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Order boundedness and weak compactness of the set of quasi-measure extensions of a quasi-measure

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Abstract: Let $\mathfrak{M}$ and $\mathfrak{R}$ be algebras of subsets of a set Ω with $\mathfrak{M} \subset \mathfrak{R}$, and denote by $E(\mu)$ the set of all quasi-measure extensions of a given quasi-measure μ on $\mathfrak{M}$ to $\mathfrak{R}$. We give some criteria for order boundedness of $E(\mu)$ in $ba(\mathfrak{R})$, in the general case as well as for atomic μ. Order boundedness implies weak compactness of $E(\mu)$. We show that the converse implication holds under some assumptions on $\mathfrak{M}$, $\mathfrak{R}$ and μ or μ alone, but not in general.

Keywords: linear lattice; order bounded; additive set function; quasi-measure; atomic; extension; convex set; extreme point; weakly compact
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References

[1] Aliprantis C.D., Burkinshaw O., *Positive Operators*, Academic Press, Orlando, 1985.
[2] Alvarez de Araya J., *A Radon–Nikodym theorem for vector and operator valued measures*, Pacific J. Math. **29** (1969), 1–10.
[3] Bhaskara Rao K. P. S., Bhaskara Rao M., *Theory of Charges. A Study of Finitely Additive Measures*, Academic Press, London, 1983.
[4] Bourbaki N., *Espaces vectoriels topologiques*, Chapitres 1 à 5, Springer, Berlin, 2007.
[5] Brooks J. K., *Weak compactness in the space of vector measures*, Bull. Amer. Math. Soc. **78** (1972), 284–287.
[6] Drewnowski L., *Topological rings of sets, continuous set functions, integration. I*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **20** (1972), 269–276.
[7] Hoffmann-Jørgensen J., *Vector measures*, Math. Scand. **28** (1971), 5–32.
[8] Lipecki Z., *On compactness and extreme points of some sets of quasi-measures and measures*, Manuscripta Math. **86** (1995), 349–365.
[9] Lipecki Z., *On compactness and extreme points of some sets of quasi-measures and measures. II*, Manuscripta Math. **89** (1996), 395–406.
[10] Lipecki Z., *Quasi-measures with finitely or countably many extreme extensions*, Manuscripta Math. **97** (1998), 469–481.
[11] Lipecki Z., *Cardinality of the set of extreme extensions of a quasi-measure*, Manuscripta Math. **104** (2001), 333–341.
[12] Lipecki Z., *The variation of an additive function on a Boolean algebra*, Publ. Math. Debrecen **63** (2003), 445–459.
[13] Lipecki Z., *On compactness and extreme points of some sets of quasi-measures and measures. III*, Manuscripta Math. **117** (2005), 463–473.
[14] Lipecki Z., *On compactness and extreme points of some sets of quasi-measures and measures. IV*, Manuscripta Math. **123** (2007), 133–146.
[15] Lipecki Z., *Cardinality of some convex sets and of their sets of extreme points*, Colloq. Math. **123** (2011), 133–147.
[16] Lipecki Z., *Compactness and extreme points of the set of quasi-measure extensions of a quasi-measure*, Dissertationes Math. (Rozprawy Mat.) **493** (2013), 59 pp.
[17] Marczewski E., *Measures in almost independent fields*, Fund. Math. **38** (1951), 217–229; reprinted in: Marczewski E., Collected Mathematical Papers, Institute of Mathematics, Polish Academy of Sciences, Warszawa, 1996, pp. 413–425.
[18] Nygaard O., Põldvere M., *Families of vector measures of uniformly bounded variation*, Arch. Math. (Basel) **88** (2007), 57–61.
[19] Plachky D., *Extremal and monogenic additive set functions*, Proc. Amer. Math. Soc. **54** (1976), 193–196.
[20] Wnuk W., *Banach Lattices with Order Continuous Norms*, PWN—Polish Sci. Publ., Warszawa, 1999.