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Antiflexible Latin directed triple systems

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Abstract: It is well known that given a Steiner triple system one can define a quasigroup operation $\cdot$ upon its base set by assigning $x \cdot x = x$ for all x and $x \cdot y = z$, where z is the third point in the block containing the pair $\{x, y\}$. The same can be done for Mendelsohn triple systems, where (x, y) is considered to be ordered. But this is not necessarily the case for directed triple systems. However there do exist directed triple systems, which induce a quasigroup under this operation and these are called Latin directed triple systems. The quasigroups associated with Steiner and Mendelsohn triple systems satisfy the flexible law $y \cdot (x \cdot y) = (y \cdot x) \cdot y$ but those associated with Latin directed triple systems need not. In this paper we study the Latin directed triple systems where the flexible identity holds for the least possible number of ordered pairs (x, y). We describe their geometry, present a surprisingly simple cyclic construction and prove that they exist if and only if the order n is $n \equiv 0$ or $1 \pmod 3$ and $n \geq 13$.

Keywords: directed triple system; quasigroup
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