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On the potential theory of some systems of coupled PDEs

Comment.Math.Univ.Carolin. 57,2 (2016) 135 –154.

Abstract: In this paper we study some potential theoretical properties of solutions and super-solutions of some PDE systems (S) of type $L_1u = -\mu_1 v$, $L_2v = -\mu_2 u$, on a domain D of $\mathbb{R}^d$, where μ_1 and μ_2 are suitable measures on D, and L_1, L_2 are two second order linear differential elliptic operators on D with coefficients of class $\mathcal{C}^\infty$. We also obtain the integral representation of the nonnegative solutions and supersolutions of the system (S) by means of the Green kernels and Martin boundaries associated with L_1 and L_2, and a convergence property for increasing sequences of solutions of (S).

Keywords: harmonic function; superharmonic function; potential; elliptic linear differential operator; kernel; coupled PDEs system; Kato measure
AMS Subject Classification: 31B05, 31B10, 31B35

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