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The σ-property in $C(X)$

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Abstract: The σ-property of a Riesz space (real vector lattice) B is: For each sequence $\{b_n\}$ of positive elements of B, there is a sequence $\{\lambda_n\}$ of positive reals, and $b \in B$, with $\lambda_n b_n \leq b$ for each n. This condition is involved in studies in Riesz spaces of abstract Egoroff-type theorems, and of the countable lifting property. Here, we examine when "σ" obtains for a Riesz space of continuous real-valued functions $C(X)$. A basic result is: For discrete X, $C(X)$ has σ iff the cardinal $|X| < \mathfrak{b}$, Rothberger's bounding number. Consequences and generalizations use the Lindelöf number $L(X)$: For a P-space X, if $L(X) \leq \mathfrak{b}$, then $C(X)$ has σ. For paracompact X, if $C(X)$ has σ, then $L(X) \leq \mathfrak{b}$, and conversely if X is also locally compact. For metrizable X, if $C(X)$ has σ, then X *is* locally compact.

Keywords: Riesz space; σ-property; bounding number; P-space; paracompact; locally compact

AMS Subject Classification: 03E17, 06F20, 46A40, 54C30, 54A25, 54D20, 54D45, 54G10

References

[BGHTZ09] Ball R., Gochev V., Hager A., Todorčević S., Zoble S., *Topological group criterion for $C(X)$ in compact-open-like topologies I.*, Topology Appl. **156** (2009), 710–720.

[BJ86] Blass A., Jech T., *On the Egoroff property of pointwise convergent sequences of functions*, Proc. Amer. Math. Society **90** (1986), 524–526.

[D74] Dodds, Theresa K.Y. Chow, *Egoroff properties and the order topology in Riesz spaces*, Trans. Amer. Math. Soc. **187** (1974), 365–375.

[D84] van Douwen E., *The integers and topology*, in Handbook of Set-theoretic Topology, North-Holland, Amsterdam, 1984, pp. 111–1676.

[E89] Engelking R., *General Topology*, Heldermann, Berlin, 1989.

[GJ60] Gillman L., Jerison M., *Rings of Continuous Functions*, The University Series in Higher Mathematics, Van Nostrand, Princeton, N.J.-Toronto-London-New York, 1960.

[HM15] Hager A., van Mill J., *Egoroff, σ, and convergence properties in some archimedean vector lattices*, Studia Math. **231** (2015), 269–285.

[HR16] Hager A., Raphael R., *The countable lifting property for Riesz space surjections*, Indag. Math., **27** (2016), 75–84.

[H68] Holbrook J., *Seminorms and the Egoroff property in Riesz spaces*, Trans. Amer. Math. Soc. **132** (1968), 67–77.

[J80] Jech T., *On a problem of L. Nachbin*, Proc. Amer. Math. Soc. **79** (1980), 341–342.

[J02] Jech T., *Set Theory*, third millennium edition, Springer, Berlin, 2003.

[LZ71] Luxemburg W.A.J., Zaanen A.C., *Riesz Spaces*, Vol. I, North-Holland Mathematical Library, North-Holland, Amsterdam-London, 1971.