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Comaximal graph of $C(X)$

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Abstract: In this article we study the comaximal graph $\Gamma'_2 C(X)$ of the ring $C(X)$. We have tried to associate the graph properties of $\Gamma'_2 C(X)$, the ring properties of $C(X)$ and the topological properties of X. Radius, girth, dominating number and clique number of the $\Gamma'_2 C(X)$ are investigated. We have shown that $2 \leq \operatorname{Rad} \Gamma'_2 C(X) \leq 3$ and if $|X| > 2$ then girth $\Gamma'_2 C(X) = 3$. We give some topological properties of X equivalent to graph properties of $\Gamma'_2 C(X)$. Finally we have proved that X is an almost P-space which does not have isolated points if and only if $C(X)$ is an almost regular ring which does not have any principal maximal ideals if and only if $\operatorname{Rad} \Gamma'_2 C(X) = 3$.

Keywords: rings of continuous functions; comaximal graph; radius; girth; dominating number; clique number; zero cellularity; P-space; almost P-space; connected space; regular ring

AMS Subject Classification: 54C40

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