

Dmitry Gavinsky<sup>1 2</sup>, Pavel Pudlák<sup>1</sup>  
*On the joint entropy of  $d$ -wise-independent variables*

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**Abstract:** How low can the joint entropy of  $n$   $d$ -wise independent (for  $d \geq 2$ ) discrete random variables be, subject to given constraints on the individual distributions (say, no value may be taken by a variable with probability greater than  $p$ , for  $p < 1$ )? This question has been posed and partially answered in a recent work of Babai [*Entropy versus pairwise independence* (preliminary version), <http://people.cs.uchicago.edu/~laci/papers/13augEntropy.pdf>, 2013]. In this paper we improve some of his bounds, prove new bounds in a wider range of parameters and show matching upper bounds in some special cases. In particular, we prove tight lower bounds for the min-entropy (as well as the entropy) of pairwise and three-wise independent balanced binary variables for infinitely many values of  $n$ .

**Keywords:**  $d$ -wise-independent variables; entropy; lower bound

**AMS Subject Classification:** 60C05

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