

Janusz Grabowski
An introduction to loopoids

Comment.Math.Univ.Carolin. 57,4 (2016) 515–526.

Abstract: We discuss a concept of *loopoid* as a non-associative generalization of Brandt groupoid. We introduce and study also an interesting class of more general objects which we call *semiloopoids*. A differential version of loopoids is intended as a framework for Lagrangian discrete mechanics.

Keywords: group; Brandt groupoid; Lie group; loop; transversals; discrete mechanics
AMS Subject Classification: Primary 20L05, 20N05, 22A22; Secondary 22E15, 22E60, 58H05

REFERENCES

- [1] Baer R., *Nets and groups*, Trans. Amer. Math. Soc. **46** (1939), 110–141.
- [2] Belousov V.D., *Foundations of the Theory of Quasigroups and Loops*, Nauka, Moscow, 1967 (in Russian).
- [3] Bruck R.H., *A Survey of Binary Systems*, Ergebnisse der Mathematik und ihrer Grenzgebiete. Neue Folge, Heft 20. Reihe: Gruppentheorie Springer, Berlin-Göttingen-Heidelberg, 1958.
- [4] Bruck R.H., *What is a loop*, in Studies in modern algebra, Studies in Mathematics, 2, Prentice-Hall, New Jersey, 1963, pp. 59–99.
- [5] Fedorov Y.N., Zenkov D.V., *Discrete nonholonomic LL systems on Lie groups*, Nonlinearity **18** (2005), 2211–2241.
- [6] Foguel T., *Groups, transversals, and loops*, Loops'99 (Prague), Comment. Math. Univ. Carolin. **41** (2000), 261–269.
- [7] Grabowska K., Grabowski J., *Variational calculus with constraints on general algebroids*, J. Phys. A **41** (2008), 175204, 25pp.
- [8] Grabowska K., Grabowski J., *Dirac algebroids in Lagrangian and Hamiltonian mechanics*, J. Geom. Phys. **61** (2011), 2233–2253.
- [9] Grabowski J., Józwickowski M., *Pontryagin maximum principle on almost Lie algebroids*, SIAM J. Control Optim. **49** (2011), 1306–1357.
- [10] Grabowski J., de Leon M., Marrero J.C., Martín de Diego D., *Nonholonomic constraints: A new viewpoint*, J. Math. Phys. **50** (2009), 013520, 17pp.
- [11] Grabowski J., Urbański P., *Lie algebroids and Poisson-Nijenhuis structures*, Rep. Math. Phys. **40** (1997), 195–208.
- [12] Grabowski J., Urbański P., *Algebroids – general differential calculi on vector bundles*, J. Geom. Phys. **31** (1999), 111–141.
- [13] Grigorian S., *G₂-structures and octonion bundles*, arXiv:1510.04226.
- [14] Iglesias D., Marrero J.C., Martín de Diego D., Martínez E., *Discrete nonholonomic Lagrangian systems on Lie groupoids*, J. Nonlinear Sci. **18** (2008), 221–276.
- [15] Iglesias D., Marrero J.C., Martín de Diego D., Padrón E., *Discrete nonholonomic in implicit form*, Discrete Contin. Dyn. Syst. **33** (2013), 1117–1135.
- [16] Kinyon M., *The coquecigrue of a Leibniz algebra*, preprint, 2003.
- [17] Kinyon M., Weinstein A., *Leibniz algebras, Courant algebroids, and multiplications on reductive homogeneous spaces*, Amer. J. Math. **123** (2001), 525–550.
- [18] Mackenzie K., *General Theory of Lie Groupoids and Lie Algebroids*, London Mathematical Society Lecture Note Series, 213, Cambridge University Press, Cambridge, 2005.
- [19] Marrero J.C., Martín de Diego D., Martínez E., *Discrete Lagrangian and Hamiltonian mechanics on Lie groupoids*, Nonlinearity **19** (2006), 1313–1348. Corrigendum: Nonlinearity **19** (2006), 3003–3004.
- [20] Marrero J.C., Martín de Diego, Stern A., *Symplectic groupoids and discrete constrained Lagrangian mechanics*, Discrete Contin. Dyn. Syst. **35** (2015), 367–397.
- [21] Martínez E., *Lagrangian mechanics on Lie algebroids*, Acta Appl. Math. **67** (2001), 295–320.
- [22] Pflugfelder H.O., *Quasigroups and Loops: Introduction*, Heldermann, Berlin, 1990.
- [23] Sabinin L.V., *Smooth Quasigroups and Loops*, Kluwer Academic Press, Dordrecht, 1999.

- [24] Sabinin L.V., *Smooth quasigroups and loops: forty-five years of incredible growth*, Comment. Math. Univ. Carolin. **41** (2000), 377–400.
- [25] Smith J.D.H., *Loops and quasigroups: Aspects of current work and prospects for the future*, Comment. Math. Univ. Carolin. **41** (2000), 415–427.
- [26] Stern A., *Discrete Hamilton–Pontryagin mechanics and generating functions on Lie groupoids*, J. Symplectic Geom. **8** (2010), 225–238.
- [27] Weinstein A., *Lagrangian mechanics and groupoids*, Fields Inst. Comm. **7** (1996), 207–231.
- [28] Zakrzewski S., *Quantum and classical pseudogroups I. Union pseudogroups and their quantization*, Comm. Math. Phys. **134** (1990), 347–370.