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## *Isotopy invariant quasigroup identities*

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**Abstract:** According to S. Krstić, there are only four quadratic varieties which are closed under isotopy. We give a simple procedure generating quadratic identities and deciding which of the four varieties they define. There are about 37000 such identities with up to five variables.

**Keywords:** quasigroup; 3-sorted quasigroup; homotopy; isotopy; quadratic identity; gemini identity; coherent identity; variety closed under isotopy (homotopy)

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### REFERENCES

- [1] Belousov V.D., *Balanced identities in quasigroups*, Mat. Sbornik **70** (112) (1966), 55–97 (Russian).
- [2] Belousov V.D., *Algebraic Nets and Quasigroups*, Štiinca, Kishinev, 1971 (Russian).
- [3] Belyavskaya G.B., *Parastrophically equivalent identities characterizing quasigroups isotopic to abelian groups*, Quasigroups Related Systems **22** (2014), no. 1, 19–32.
- [4] Birkhoff G., Lipson J.D., *Heterogeneous algebras*, J. Combinatorial Theory **8** (1970), 115–133.
- [5] Brožíková E., *On universal quasigroup identities*, Math. Bohem. **117** (1992), no. 1, 20–32.
- [6] Drápal A., *On multiplication groups of relatively free quasigroups isotopic to abelian groups*, Czechoslovak Math. J. **55(130)** (2005), 61–87.
- [7] Falconer E., *Isotopy invariants in quasigroups*, Trans. Amer. Math. Soc. **151** (1970), no. 2, 511–526.
- [8] Gvaramiya A.A., *Automata and quasigroups*, Soobshch. Akad. Nauk Gruzin. SSR **114** (1984), no. 3, 481–484 (Russian).
- [9] Gvaramiya A.A., *Quasivarieties of automata. Connections with quasigroups*, Sibirsk. Mat. Zh. **26** (1985), no. 3, 11–30 (Russian); <http://dx.doi.org/10.1007/BF00968618>; English translation: Siberian Math. J. **26** (1985), 315–331, DOI: 10.1007/BF00968618
- [10] Gvaramiya A.A., *Quasigroup classes that are invariant under isotopy, and abstract classes of invertible automata*, Dokl. Akad. Nauk SSSR **282** (1985), no. 5, 1047–1051 (Russian); English translation: Soviet. Math. Dokl. **31** (1985), 545–549.
- [11] Gvaramiya A.A., *Category of quasigroups with homotopies*, C.R. Acad. Bulgare Sci. **38** (1985), no. 6, 663–666.
- [12] Gvaramiya A.A., Plotkin B.I., *The homotopies of quasigroups and universal algebras*, in A. Romanowska, J.D.H. Smith (eds.), Universal Algebra and Quasigroup Theory (Jadwisin, 1989), Res. Expo. Math., 19, Heldermann, Berlin, 1992, pp. 89–99.
- [13] Krapež A., Taylor M.A., *Gemini functional equations on quasigroups*, Publ. Math. Debrecen **47** (1995), no. 3–4, 281–292.
- [14] Krstić S., *Quadratic quasigroup identities*, PhD thesis (Serbocroatian), University of Belgrade, 1985, <http://eLibrary.matf.bg.ac.rs/handle/123456789/182/phdSavaKrstic.pdf>, (accessed May 5, 2015).
- [15] Krstić S., *On quasigroup varieties closed under isotopies*, Publ. Inst. Math. (Beograd) (N.S.) **39(53)** (1986), 89–95.
- [16] Manzano M., *Extensions of First Order Logic*, Cambridge Tracts in Theoretical Computer Science, 19, Cambridge University Press, Cambridge, New York, 2005.
- [17] Milić S., *On GD-groupoids with applications to n-ary quasigroups*, Publ. Inst. Math. (Beograd) (N.S.) **13(27)** (1972), 65–76.
- [18] Petrescu A., *Certain questions of the theory of homotopy of universal algebras*, in Contributions to universal algebra (Colloq. Math. Soc. Janos Boylai Vol. 17), North Holland, Amsterdam, 1977, pp. 341–355.
- [19] Satyabhama V., *Generalized distributivity equation on GD-groupoids*, Mat. Vesnik **29** (1977), no. 1, 137–146.
- [20] Shahbapour Kh., *An hyperidentity concerning the quasigroup identities isotopy closure to abelian group*, Austral. J. Basic Appl. Sci. **3(4)**, (2009).

- [21] Sioson F.M., *On generalized algebras*, Portugal. Math. **25** (1966), 67–90.
- [22] Sokhatskii F.N., *On isotopes of groups II*, Ukrain. Mat. Zh. **47** (1995), no. 12, 1692–1703 (Ukrainian); English translation: Ukrainian Math. J. **47** (1995), no. 12, 1935–1948.
- [23] Stojaković Z., *Generalized entropy on GD-groupoids with application to quasigroups of various arities*, Mat. Vesnik **24** (1972), no. 1, 159–166.
- [24] Voutsadakis G., *Categorical models and quasigroup homotopies*, Theory Appl. Categ. **11** (2003), no. 1, 1–14.