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Quantum idempotence, distributivity, and the Yang-Baxter equation

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Abstract: Quantum quasigroups and loops are self-dual objects that provide a general framework for the nonassociative extension of quantum group techniques. They also have one-sided analogues, which are not self-dual. In this paper, natural quantum versions of idempotence and distributivity are specified for these and related structures. Quantum distributive structures furnish solutions to the quantum Yang-Baxter equation.

Keywords: Hopf algebra; quantum group; quasigroup; loop; quantum Yang-Baxter equation; distributive
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