S. Ebrahimi Atani, S. Dolati Pish Hesari, M. Khoramdel

A graph associated to proper non-small ideals of a commutative ring

Comment.Math.Univ.Carolin. 58,1 (2017) 1 –12.

Abstract: In this paper, a new kind of graph on a commutative ring is introduced and investigated. Small intersection graph of a ring R, denoted by $G(R)$, is a graph with all non-small proper ideals of R as vertices and two distinct vertices I and J are adjacent if and only if $I \cap J$ is not small in R. In this article, some interrelation between the graph theoretic properties of this graph and some algebraic properties of rings are studied. We investigated the basic properties of the small intersection graph as diameter, girth, clique number, cut vertex, planar property and independence number. Further, it is shown that the independence number of a small graph of a ring R is equal to the number of its maximal ideals and the domination number of small graph is at most 2.

Keywords: small ideal; small intersection graph; clique number; independence number; domination number; planar property
AMS Subject Classification: 05C40, 05C25, 13A15

References

[1] Akbari S., Tavallaee H.A., Ghezelahmad S. Khalashi, *Intersection graph of submodules of a module*, J. Algebra Appl. **11** (2012), 1250019.
[2] Anderson D.F., Livingston P.S., *The zero-divisor graph of a commutative rings*, J. Algebra **217** (1999), 434–447.
[3] Beck I., *Coloring of commutative rings*, J. Algebra **116** (1988), 208–226.
[4] Bondy J.A., Murty U.S.R., *Graph Theory*, Graduate Texts in Mathematics, 244, Springer, New York, 2008.
[5] Bosak J., *The graphs of semigroups*, in Theory of Graphs and its Applications, Academic Press, New York, 1964, pp. 119–125.
[6] Chakrabarty I., Ghosh S., Mukherjee T.K., Sen M.K., *Intersection graphs of ideals of rings*, Discrete Math. **309** (2009), 5381–5392.
[7] Chelvam T., Asir T., *The intersection graph of gamma sets in the total graph I*, J. Algebra Appl. **12** (2013), no. 4, 1250198, 18 pp.
[8] Csákány B., Pollák G., *The graph of subgroups of a finite group*, Czechoslovak Math. J. **19** (1969), 241–247.
[9] Haynes T.W., Hedetniemi S.T., Slater P.J. (eds.), *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York, NY, 1998.
[10] Jafari S.H., Jafari Rad N., *Domination in the intersection graphs of rings and modules*, Ital. J. Pure Appl. Math. **28** (2011), 19–22.
[11] Lam T.Y., *A First Course in Non-commutative Rings*, Graduate Texts in Mathematics, 131, Springer, Berlin-Heidelberg-New York, 1991.
[12] Pucanović Z.S., Radovanović M., Aleksandra E.L., *On the genus of the intersection graph of ideals of a commutative ring*, J. Algebra Appl. **13** (2014), no. 5, 1350155, 20 pp.
[13] Pucanović Z.S., Petrović Z.Z., *Toroidality of intersection graphs of ideals of commutative rings*, Graphs Combin. **30** (2014), no. 3, 707–716.
[14] Yaraneri E., *Intersection graph of a module*, J. Algebra Appl. **12** (2013), no. 5, 1250218, 30 pp.
[15] Wisbauer R., *Foundations of Module and Ring Theory*, Gordon and Breach Science Publishers, Philadelphia, 1991.