Ioana Ghenciu
The weak Gelfand-Phillips property in spaces of compact operators

Abstract: For Banach spaces X and Y, let $K_{w^*}(X^*, Y)$ denote the space of all $w^* - w$ continuous compact operators from X^* to Y endowed with the operator norm. A Banach space X has the wGP property if every Grothendieck subset of X is relatively weakly compact. In this paper we study Banach spaces with property wGP. We investigate whether the spaces $K_{w^*}(X^*, Y)$ and $X \otimes_\epsilon Y$ have the wGP property, when X and Y have the wGP property.

References

[1] Bourgain J., *New Classes of $\mathcal{L}_p$-spaces*, Lecture Notes in Mathematics, 889, Springer, Berlin-New York, 1981.

[2] Bourgain J., Diestel J., *Limited operators and strict cosingularity*, Math. Nachr. **119** (1984), 55–58.

[3] Bessaga C., Pelczynski A., *On bases and unconditional convergence of series in Banach spaces*, Studia Math. **17** (1958), 151–174.

[4] Collins H.S., Ruess W., *Weak compactness in the space of compact operators of vector valued functions*, Pacific J. Math. **106** (1983), 45–71.

[5] Diestel J., *Sequences and Series in Banach Spaces*, Graduate Texts in Mathematics, 92, Springer, Berlin, 1984.

[6] Diestel J., *A survey of results related to the Dunford-Pettis property*, Contemporary Math. **2** (1980), 15–60.

[7] Diestel J., Uhl J.J., Jr., *Vector measures*, Math. Surveys, 15, American Mathematical Society, Providence, RI, 1977.

[8] Cembranos P., Mendoza J., *Banach Spaces of Vector-Valued Functions*, Lecture Notes in Mathematics, 1676, Springer, Berlin, 1997.

[9] Domanski P., Lindstrom M., Schluchtermann G., *Grothendieck operators on tensor products*, Proc. Amer. Math. Soc. **125** (1997), 2285–2291.

[10] Drewnowski L., *On Banach spaces with the Gelfand-Phillips property*, Math. Z. **193** (1986), 405–411.

[11] Drewnowski L., Emmanuele G., *On Banach spaces with the Gelfand-Phillips property II*, Rend. Circolo Mat. Palermo **38** (1989), 377–391.

[12] Emmanuele G., *Banach spaces in which Dunford-Pettis sets are relatively compact*, Arch. Math. **58** (1992), 477–485.

[13] Emmanuele G., *The (BD) property in $L^1(\mu, E)$*, Indiana Univ. Math. J. **36** (1987), 229–230.

[14] Emmanuele G., *A dual characterization of Banach spaces not containing ℓ^1*, Bull. Polish Acad. Sci. Math. **34** (1986), 155–160.

[15] Fabian M., Habla P., Hájek P., Montesinos V., Zizler V., *Banach Space Theory. The Basis for Linear and Nonlinear Analysis*, CMS Books in Mathematics, Springer, New York, 2011.

[16] Fabian M.J., *Gâteaux Differentiability of Convex Functions and Topology. Weak Asplund Spaces*, Canad. Math. Soc. Series of Monographs and Advanced Texts, A Wiley-Interscience Publication, John Wiley & Sons, Inc., New York, 1997.

[17] Ghenciu I., *Weak precompactness and property (V^*)*, Colloq. Math. **138** (2014), no. 2, 255–269.

[18] Ghenciu I., Lewis P., *Completely continuous operators*, Colloq. Math. **126** (2012), no. 2, 231–256, doi:10.4064/cm126-2-7.

[19] Ghenciu I., Lewis P., *The embeddability of c_0 in spaces of operators*, Bull. Polish. Acad. Sci. Math. **56** (2008), 239–256.

[20] Ghenciu I., Lewis P., *Almost weakly compact operators*, Bull. Polish. Acad. Sci. Math. **54** (2006), 237–256.

2

[21] Hagler J., Odell E., *A Banach space not containing ℓ_1, whose dual ball is not weak* sequentially compact*, Illinois J. Math **22** (1978), 290–294.

[22] Haydon R., Levy M., Odell E., *On sequences without weak* convergent convex block subsequences*, Proc. Amer. Soc. **101** (1987), 94–98.

[23] Leung D.H., *A Gelfand-Phillips property with respect to the weak topology*, Math. Nachr. **149** (1990), 177–181.

[24] Lindenstrauss J., *On nonseparable reflexive Banach spaces*, Bull. Amer. Math. Soc. **72** (1966), 967–970.

[25] Lindenstrauss J., Tzafriri L., *Classical Banach Spaces II*, Ergebnisse der Mathematik und ihrer Grenzgebiete, 97, Springer, Berlin-Heidelberg-New York, 1979.

[26] Pełczyński A., *On Banach spaces containing $L^1(\mu)$*, Studia Math. **30** (1968), 231–246.

[27] Pełczyński A., *Banach spaces on which every unconditionally converging operator is weakly compact*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **10** (1962), 641–648.

[28] Pełczyński A., Semadeni Z., *Spaces of continuous functions III*, Studia Math. **18** (1959), 211–222.

[29] Ruess W., *Duality and geometry of spaces of compact operators*, Functional Analysis: Surveys and Recent Results III. Proc. 3rd Paderborn Conference 1983, North-Holland Math. Studies, 90, North-Holland, Amsterdam, 1984, pp. 59–78.

[30] Ryan R.A., *Intoduction to Tensor Products of Banach Spaces*, Springer, London, 2002.

[31] Schlumprecht T., *Limited sets in Banach spaces*, Dissertation, Munich, 1987.

[32] Ülger A., *Continuous linear operators on $C(K, X)$ and pointwise weakly precompact subsets of $C(K, X)$*, Math. Proc. Cambridge Philos. Soc. **111** (1992), 143–150.