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Several quantitative characterizations of some specific groups

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Abstract: Let G be a finite group and let $\pi(G) = \{p_1, p_2, \ldots, p_k\}$ be the set of prime divisors of $|G|$ for which $p_1 < p_2 < \cdots < p_k$. The Gruenberg-Kegel graph of G, denoted $\mathrm{GK}(G)$, is defined as follows: its vertex set is $\pi(G)$ and two different vertices p_i and p_j are adjacent by an edge if and only if G contains an element of order p_ip_j. The degree of a vertex p_i in $\mathrm{GK}(G)$ is denoted by $d_G(p_i)$ and the k-tuple $D(G) = (d_G(p_1), d_G(p_2), \ldots, d_G(p_k))$ is said to be the degree pattern of G. Moreover, if $\omega \subseteq \pi(G)$ is the vertex set of a connected component of $\mathrm{GK}(G)$, then the largest ω-number which divides $|G|$, is said to be an order component of $\mathrm{GK}(G)$. We will say that the problem of OD-characterization is solved for a finite group if we find the number of pairwise non-isomorphic finite groups with the same order and degree pattern as the group under study. The purpose of this article is twofold. First, we completely solve the problem of OD-characterization for every finite non-abelian simple group with orders having prime divisors at most 29. In particular, we show that there are exactly two non-isomorphic finite groups with the same order and degree pattern as $U_4(2)$. Second, we prove that there are exactly two non-isomorphic finite groups with the same order components as $U_5(2)$.

Keywords: OD-characterization of finite group; prime graph; degree pattern; simple group; 2-Frobenius group
AMS Subject Classification: 20D05, 20D06, 20D08

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