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Elementary stochastic calculus for finance with infinitesimals

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Abstract: The concept of an equivalent martingale measure is of key importance for pricing of financial derivative contracts. The goal of the paper is to apply infinitesimals in the non-standard analysis set-up to provide an elementary construction of the equivalent martingale measure built on hyperfinite binomial trees with infinitesimal time steps.

Keywords: equivalent martingale measure; option pricing; stochastic processes; non-standard analysis

AMS Subject Classification: Primary 60H05, 60J65

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