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On a bifurcation problem arising in cholesteric liquid crystal theory

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Abstract: In a cholesteric liquid crystal the director field $n(x, y, z)$ tends to form a right-angle helicoid around a twist axis in order to minimize the internal energy; however, a fixed alignment of the director field at the boundary (strong anchoring) can give rise to distorted configurations of the director field, as oblique helicoid, in order to save energy. The transition to this distorted configurations depend on the boundary conditions and on the geometry of the liquid crystal, and it is known as Freedericksz transition (without external fields). We consider the classical situation of a thin layer between two glass sheet assuming the Oseen-Frank model for the energy, and that the director field depend only on the direction z orthogonal to the layer; then we focus on two kinds of boundary conditions: the planar case and the orthogonal case. In the first, we impose that $n(0) = (1, 0, 0)$, $n(d) = (\cos\alpha, \sin\alpha, 0)$ (where $z = 0$ and $z = d > 0$ are, respectively, the bottom and the top of the layer), and search for the couples (d, α) such that oblique helicoid appear. In the case $K_1 > 0$, $K_2 = K_3 = 1$ for the elastic constants of the Oseen-Frank energy, we completely characterize these couples. In the second case it is a classical result that oblique helicoid bifurcates from the trivial solution $n(z) = (0, 0, 1)$ for suitable values of d; then we study the exact number of these nontrivial solutions and their stability.

Keywords: bifurcation; time map; cholesteric liquid crystals; Freedericksz transition
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