

**Ondřej F.K. Kalenda**  
*On projectional skeletons in Vašák spaces*

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**Abstract:** We provide an alternative proof of the theorem saying that any Vašák (or, weakly countably determined) Banach space admits a full 1-projectional skeleton. The proof is done with the use of the method of elementary submodels and is comparably simple as the proof given by W. Kubiś (2009) in case of weakly compactly generated spaces.

**Keywords:** Vašák Banach space; projectional skeleton; elementary submodel

**AMS Subject Classification:** 46B26, 03C30

REFERENCES

- [1] Amir D., Lindenstrauss J., *The structure of weakly compact sets in Banach spaces*, Ann. of Math. **88** (1968), no. 2, 35–46.
- [2] Bohata M., Hamhalter J., Kalenda O., *Decompositions of preduals of JBW and JBW\* algebras*, J. Math. Anal. Appl. **446** (2017), no. 1, 18–37.
- [3] Cúth M., *Separable reduction theorems by the method of elementary submodels*, Fund. Math. **219** (2012), no. 3, 191–222.
- [4] Cúth M., Kalenda O.F.K., *Rich families and elementary submodels*, Cent. Eur. J. Math. **12** (2014), no. 7, 1026–1039.
- [5] Cúth M., Kalenda O.F.K., *Monotone retractability and retractional skeletons*, J. Math. Anal. Appl. **423** (2015), no. 1, 18–31.
- [6] Cúth M., Rmoutil M., Zelený M., *On separable determination of  $\sigma$ -P-porous sets in Banach spaces*, Topology Appl. **180** (2015), 64–84.
- [7] Dow A., *An introduction to applications of elementary submodels to topology*, Topology Proc. **13** (1988), no. 1, 17–72.
- [8] Fabian M., Godefroy G., *The dual of every Asplund space admits a projectional resolution of the identity*, Studia Math. **91** (1988), no. 2, 141–151.
- [9] Fabian M.J., *Gâteaux Differentiability of Convex Functions and Topology. Weak Asplund Spaces*, Canadian Mathematical Society Series of Monographs and Advanced Texts, A Wiley-Interscience Publication, John Wiley & Sons, Inc., New York, 1997.
- [10] Gul’ko S.P., *The structure of spaces of continuous functions and their hereditary paracompactness*, Uspekhi Mat. Nauk **34** (1979), no. 6(210), 33–40.
- [11] Kechris A.S., *Classical Descriptive Set Theory*, Graduate Texts in Mathematics, 156, Springer, New York, 1995.
- [12] Koszmider P., *Projections in weakly compactly generated Banach spaces and Chang’s conjecture*, J. Appl. Anal. **11** (2005), no. 2, 187–205.
- [13] Kubiś W., *Banach spaces with projectional skeletons*, J. Math. Anal. Appl. **350** (2009), no. 2, 758–776.
- [14] Kunen K., *Set Theory. An Introduction to Independence Proofs*, reprint of the 1980 original, Studies in Logic and the Foundations of Mathematics, 102, North-Holland Publishing Co., Amsterdam, 1983.
- [15] Mercourakis S., *On weakly countably determined Banach spaces*, Trans. Amer. Math. Soc. **300** (1987), no. 1, 307–327.
- [16] Orihuela J., Valdivia M., *Projective generators and resolutions of identity in Banach spaces*, Congress on Functional Analysis (Madrid, 1988), Rev. Mat. Univ. Complut. Madrid 2 (1989), suppl., 179–199.
- [17] Valdivia M., *Resolutions of the identity in certain Banach spaces*, Collect. Math. **39** (1988), no. 2, 127–140.
- [18] Valdivia M., *Simultaneous resolutions of the identity operator in normed spaces*, Collect. Math. **42** (1991), no. 3, 265–284 (1992).
- [19] Vašák L., *On one generalization of weakly compactly generated Banach spaces*, Studia Math. **70** (1981), no. 1, 11–19.