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The prime ideals intersection graph of a ring

Comment.Math.Univ.Carolin. 58,2 (2017) 137–145.

Abstract: Let R be a commutative ring with unity and $U(R)$ be the set of unit elements of R . In this paper, we introduce and investigate some properties of a new kind of graph on the ring R , namely, the prime ideals intersection graph of R , denoted by $G_p(R)$. The $G_p(R)$ is a graph with vertex set $R^* - U(R)$ and two distinct vertices a and b are adjacent if and only if there exists a prime ideal \mathfrak{p} of R such that $a, b \in \mathfrak{p}$. We obtain necessary and sufficient conditions on R such that $G_p(R)$ is disconnected. We find the diameter and girth of $G_p(R)$. We also determine all rings whose prime ideals intersection graph is a star, path, or cycle. At the end of this paper, we study the planarity and outerplanarity of $G_p(R)$.

Keywords: the prime ideals intersection graph of a ring; clique number; planar graph

AMS Subject Classification: 05C40, 05C69, 13A15

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