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On the structure of universal differentiability sets

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Abstract: A subset of $\mathbb{R}^d$ is called a universal differentiability set if it contains a point of differentiability of every Lipschitz function $f\colon \mathbb{R}^d \to \mathbb{R}$. We show that any universal differentiability set contains a 'kernel' in which the points of differentiability of each Lipschitz function are dense. We further prove that no universal differentiability set may be decomposed as a countable union of relatively closed, non-universal differentiability sets.

Keywords: differentiability; Lipschitz functions; universal differentiability set; σ-porous set

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