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Symmetric linear operator identities in quasigroups

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Abstract: Let G be a quasigroup. Associativity of the operation on G can be expressed by the symbolic identity $R_x L_y = L_y R_x$ of left and right multiplication maps; likewise, commutativity can be expressed by the identity $L_x = R_x$. In this article, we investigate symmetric linear identities: these are identities in left and right multiplication symbols in which every indeterminate appears exactly once on each side, and whose sides are mirror images of each other. We determine precisely which identities imply associativity and which imply commutativity, providing counterexamples as appropriate. We apply our results to show that there are exactly eight varieties of quasigroups satisfying such identities, and determine all inclusion relations among them.

Keywords: quasigroup; linear identity; associativity; commutativity

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