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Linear extenders and the Axiom of Choice

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Abstract: In set theory without the Axiom of Choice **ZF**, we prove that for every commutative field $\mathbb{K}$, the following statement $\mathbf{D}_{\mathbb{K}}$: "On every non null $\mathbb{K}$-vector space, there exists a non null linear form" implies the existence of a "$\mathbb{K}$-linear extender" on every vector subspace of a $\mathbb{K}$-vector space. This solves a question raised in Morillon M., *Linear forms and axioms of choice*, Comment. Math. Univ. Carolin. **50** (2009), no. 3, 421-431. In the second part of the paper, we generalize our results in the case of spherically complete ultrametric valued fields, and show that Ingleton's statement is equivalent to the existence of "isometric linear extenders".

Keywords: Axiom of Choice; extension of linear forms; non-Archimedean fields; Ingleton's theorem
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