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Resolvability in c.c.c. generic extensions

Comment.Math.Univ.Carolin. 58,4 (2017) 519 –529.

Abstract: Every crowded space X is ω-resolvable in the c.c.c. generic extension $V^{\mathrm{Fn}(|X|,2)}$ of the ground model. We investigate what we can say about λ-resolvability in c.c.c. generic extensions for $\lambda > \omega$. A topological space is *monotonically ω_1-resolvable* if there is a function $f : X \to \omega_1$ such that

$$\{x \in X : f(x) \geq \alpha\} \subset^{dense} X$$

for each $\alpha < \omega_1$. We show that given a T_1 space X the following statements are equivalent: (1) X is ω_1-resolvable in some c.c.c. generic extension; (2) X is monotonically ω_1-resolvable; (3) X is ω_1-resolvable in the Cohen-generic extension $V^{\mathrm{Fn}(\omega_1,2)}$. We investigate which spaces are monotonically ω_1-resolvable. We show that if a topological space X is c.c.c., and $\omega_1 \leq \Delta(X) \leq |X| < \omega_\omega$, where $\Delta(X) = \min\{|G| : G \neq \emptyset \text{ open}\}$, then X is monotonically ω_1-resolvable. On the other hand, it is also consistent, modulo the existence of a measurable cardinal, that there is a space Y with $|Y| = \Delta(Y) = \aleph_\omega$ which is not monotonically ω_1-resolvable. The characterization of ω_1-resolvability in c.c.c. generic extension raises the following question: is it true that crowded spaces from the ground model are ω-resolvable in $V^{\mathrm{Fn}(\omega,2)}$? We show that (i) if $V = L$ then every crowded c.c.c. space X is ω-resolvable in $V^{\mathrm{Fn}(\omega,2)}$, (ii) if there are no weakly inaccessible cardinals, then every crowded space X is ω-resolvable in $V^{\mathrm{Fn}(\omega_1,2)}$. Moreover, it is also consistent, modulo a measurable cardinal, that there is a crowded space X with $|X| = \Delta(X) = \omega_1$ such that X remains irresolvable after adding a single Cohen real.

Keywords: resolvable; monotonically ω_1-resolvable; measurable cardinal
AMS Subject Classification: 54A35, 03E35, 54A25

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