Salvatore Bonafede

Hölder continuity of bounded generalized solutions for some degenerated quasilinear elliptic equations with natural growth terms

Comment.Math.Univ.Carolin. 59,1 (2018) 45 –64.

Abstract: We prove the local Hölder continuity of bounded generalized solutions of the Dirichlet problem associated to the equation

$$\sum_{i=1}^{m} \frac{\partial}{\partial x_i} a_i(x,u,\nabla u) - c_0|u|^{p-2}u = f(x,u,\nabla u),$$

assuming that the principal part of the equation satisfies the following degenerate ellipticity condition

$$\lambda(|u|) \sum_{i=1}^{m} a_i(x,u,\eta)\eta_i \geq \nu(x)|\eta|^p,$$

and the lower-order term f has a natural growth with respect to ∇u.

Keywords: elliptic equations; weight function; regularity of solutions
AMS Subject Classification: 35J15, 35J70, 35B65

References

[1] Bensoussan A., Boccardo L., Murat F., *On a nonlinear partial differential equation having natural growth terms and unbounded solution*, Ann. Inst. Henri Poincaré **5** (1988), no. 4, 347–364.

[2] Boccardo L., Murat F., Puel J. P., *Existence de solutions faibles pour des équations elliptiques quasi-linéares à croissance quadratique*, Nonlinear Partial Differential Equations and Their Applications, College de France Seminar, Vol. IV, Res. Notes in Math., 84, Pitman, London, 1983, 19–73 (French. English summary).

[3] Boccardo L., Murat F., Puel J. P., *Résultat d'existence pour certains problèmes elliptiques quasilinéaires*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **11** (1984), no. 2, 213–235 (French).

[4] Bonafede S., *Quasilinear degenerate elliptic variational inequalities with discontinuous coefficients*, Comment. Math. Univ. Carolin. **34** (1993), no. 1, 55–61.

[5] Bonafede S., *Existence and regularity of solutions to a system of degenerate nonlinear elliptic equations*, Br. J. Math. Comput. Sci. **18** (2016), no. 5, 1–18.

[6] Bonafede S., *Existence of bounded solutions of Neumann problem for a nonlinear degenerate elliptic equation*, Electron. J. Differential Equations **2017** (2017), no. 270, 1–21.

[7] Cirmi G. R., D'Asero S., Leonardi S., *Fourth-order nonlinear elliptic equations with lower order term and natural growth conditions*, Nonlinear Anal. **108** (2014), 66–86.

[8] Del Vecchio T., *Strongly nonlinear problems with hamiltonian having natural growth*, Houston J. Math. **16** (1990), no. 1, 7–24.

[9] Drábek P., Nicolosi F., *Existence of bounded solutions for some degenerated quasilinear elliptic equations*, Ann. Mat. Pura Appl. **165** (1993), 217–238.

[10] Fabes E. B., Kenig C. E., Serapioni R. P., *The local regularity of solutions of degenerate elliptic equations*, Comm. Partial Differential Equations **7** (1982), 77–116.

[11] Gilbarg D., Trudinger N. S., *Elliptic Partial Differential Equations of Second Order*, Springer, Berlin, 1983.

[12] Guglielmino F., Nicolosi F., *W-solutions of boundary value problems for degenerate elliptic operators*, Ricerche Mat. **36** (1987), suppl., 59–72.

[13] Guglielmino F., Nicolosi F., *Existence theorems for boundary value problems associated with quasilinear elliptic equations*, Ricerche Mat. **37** (1988), 157–176.

[14] John F., Nirenberg L., *On functions of bounded mean oscillation*, Comm. Pure Appl. Math. **14** (1961), 415–426.

[15] Kovalevsky A., Nicolosi F., *Boundedness of solutions of variational inequalities with nonlinear degenerated elliptic operators of high order*, Appl. Anal. **65** (1997), 225–249.

[16] Kovalevsky A., Nicolosi F., *On Hölder continuity of solutions of equations and variational inequalities with degenerate nonlinear elliptic high order operators*, Current Problems of Analysis and Mathematical Physics, Taormina 1998, Aracne, Rome, 2000, 205–220

[17] Kovalevsky A., Nicolosi F., *Boundedness of solutions of degenerate nonlinear elliptic variational inequalities*, Nonlinear Anal. **35** (1999), 987–999.

[18] Kovalevsky A., Nicolosi F., *On regularity up to the boundary of solutions to degenerate nonlinear elliptic high order equations*, Nonlinear Anal. **40** (2000), 365–379.

[19] Ladyzhenskaya O., Ural'tseva N., *Linear and Quasilinear Elliptic Equations*, translated from the Russian, Academic Press, New York-London, 1968.

[20] Landes R., *Solvability of perturbed elliptic eqautions with critical growth exponent for the gradient*, J. Math. Anal. Appl. **139** (1989), 63–77.

[21] Moser J., *A new proof of De Giorgi's theorem concerning the regularity problem for elliptic differential equations*, Comm. Pure Appl. Math. **13** (1960), pp. 457–468.

[22] Murthy M. K. V., Stampacchia G., *Boundary value problems for some degenarate elliptic operators*, Ann. Mat. Pura Appl. (4) **80** (1968), 1–122.

[23] Serrin J. B., *Local behavior of solutions of quasi-linear equations*, Acta Math. **111** (1964), 247–302.

[24] Skrypnik I. V., *Nonlinear Higher Order Elliptic Equations*, Naukova dumka, Kiev, 1973 (Russian).

[25] Skrypnik I. V., *Higher order quasilinear elliptic equations with continuous generalized solutions*, Differ. Equ. **14** (1978), no. 6, 786–795.

[26] Trudinger N. S., *On Harnack type inequalities and their application to quasilinear elliptic equations*, Comm. Pure Appl. Math. **20** (1967), 721–747.

[27] Trudinger N. S., *On the regularity of generalized solutions of linear non-uniformly elliptic equations*, Arch. Ration. Mech. Anal. **42** (1971), 51–62.

[28] Trudinger N. S., *Linear elliptic operators with measurable coefficients*, Ann. Scuola Norm. Sup. Pisa **27** (1973), 265–308.

[29] Voitovich M. V., *Existence of bounded solutions for a class of nonlinear fourth-order equations*, Differ. Equ. Appl. **3** (2011), no. 2, 247–266.

[30] Voitovich M. V., *Existence of bounded solutions for nonlinear fourth-order elliptic equations with strengthened coercivity and lower-terms with natural growth*, Electron. J. Differential Equations **2013** (2013), no. 102, 25 pages.

[31] Voitovich M. V., *On the existence of bounded generalized solutions of the Dirichlet problem for a class of nonlinear high-order elliptic equations*, J. Math. Sci. (N.Y.) **210** (2015), no. 1, 86–113.

[32] Voitovych M. V., *Hölder continuity of bounded generalized solutions for nonlinear fourth-order elliptic equations with strengthened coercivity and natural growth terms*, Electron. J. Differential Equations **2017** (2017), no. 63, 18 pages.

[33] Zamboni P., *Hölder continuity for solutions of linear degenerate elliptic equations under minimal assumptions*, J. Differential Equations **182** (2002), 121–140.