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Some versions of second countability of metric spaces in ZF and their role to compactness

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Abstract: In the realm of metric spaces we show in ZF that: (i) A metric space is compact if and only if it is countably compact and for every $\varepsilon > 0$, every cover by open balls of radius ε has a countable subcover. (ii) Every second countable metric space has a countable base consisting of open balls if and only if the axiom of countable choice restricted to subsets of $\mathbb{R}$ holds true. (iii) A countably compact metric space is separable if and only if it is second countable.

Keywords: axiom of choice; compact space; countably compact space; totally bounded space; Lindelöf space; separable space, second countable metric space
AMS Subject Classification: 54E35, 54E45

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