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Some results on the co-intersection graph of submodules of a module

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Abstract: Let R be a ring with identity and M be a unitary left R-module. The co-intersection graph of proper submodules of M, denoted by $\Omega(M)$, is an undirected simple graph whose vertex set $V(\Omega)$ is a set of all nontrivial submodules of M and two distinct vertices N and K are adjacent if and only if $N + K \neq M$. We study the connectivity, the core and the clique number of $\Omega(M)$. Also, we provide some conditions on the module M, under which the clique number of $\Omega(M)$ is infinite and $\Omega(M)$ is a planar graph. Moreover, we give several examples for which n the graph $\Omega(\mathbb{Z}_n)$ is connected, bipartite and planar.

Keywords: co-intersection graph; core; clique number; planarity
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