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Invariant symbolic calculus for semidirect products

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Abstract: Let G be the semidirect product $V \rtimes K$ where K is a connected semisimple non-compact Lie group acting linearly on a finite-dimensional real vector space V. Let π be a unitary irreducible representation of G which is associated by the Kirillov-Kostant method of orbits with a coadjoint orbit of G whose little group is a maximal compact subgroup of K. We construct an invariant symbolic calculus for π, under some technical hypothesis. We give some examples including the Poincaré group.

Keywords: semidirect products; invariant symbolic calculus; coadjoint orbit; unitary representation; Berezin quantization; Weyl quantization; Poincaré group
AMS Subject Classification: 81S10, 22E46, 22E45, 22D30, 81R05

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