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Making holes in the cone, suspension and hyperspaces of some continua

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Abstract: A connected topological space Z is *unicoherent* provided that if $Z = A \cup B$ where A and B are closed connected subsets of Z , then $A \cap B$ is connected. Let Z be a unicoherent space, we say that $z \in Z$ makes a hole in Z if $Z - \{z\}$ is not unicoherent. In this work the elements that make a hole to the cone and the suspension of a metric space are characterized. We apply this to give the classification of the elements of hyperspaces of some continua that make them hole.

Keywords: continuum; hyperspace; hyperspace suspension; property (b); unicoherence; cone; suspension

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