

Alireza K. Asboei, Seyed S. S. Amiri
The small Ree group ${}^2G_2(3^{2n+1})$ and related graph

Comment.Math.Univ.Carolin. 59,3 (2018) 271–276.

Abstract: Let G be a finite group. The main supergraph $\mathcal{S}(G)$ is a graph with vertex set G in which two vertices x and y are adjacent if and only if $o(x) \mid o(y)$ or $o(y) \mid o(x)$. In this paper, we will show that $G \cong {}^2G_2(3^{2n+1})$ if and only if $\mathcal{S}(G) \cong \mathcal{S}({}^2G_2(3^{2n+1}))$. As a main consequence of our result we conclude that Thompson’s problem is true for the small Ree group ${}^2G_2(3^{2n+1})$.

Keywords: main supergraph; simple Ree group; Thompson’s problem

AMS Subject Classification: 20D08, 05C25

REFERENCES

- [1] Asboei A.K., Amiri S.S.S., *Some alternating and symmetric groups and related graphs*, Beitr. Algebra Geom. **59** (2018), no. 1, 21–24.
- [2] Asboei A.K., Amiri S.S.S., *Some results on the main supergraph of finite groups*, accepted in Algebra Discrete Math.
- [3] Cameron P.J., *The power graph of a finite group II*, J. Group Theory **13** (2010), no. 6, 779–783.
- [4] Chakrabarty I., Ghosh S., Sen M.K., *Undirected power graphs of semigroups*, Semigroup Forum **78** (2009), no. 3, 410–426.
- [5] Chen G.Y., *On structure of Frobenius group and 2-Frobenius group*, J. Southwest China Normal Univ. **20** (1995), no. 5, 485–487 (Chinese).
- [6] Ebrahimzadeh B., Iranmanesh A., Parvizi Mosaed H., *A new characterization of Ree group ${}^2G_2(q)$ by the order of group and the number of elements*, Int. J. Group Theory **6** (2017), no. 4, 1–6.
- [7] Frobenius G., *Verallgemeinerung des Sylow’schen Satzes*, Berl. Ber. (1895), 981–993 (German).
- [8] Hamzeh A., Ashrafi A.R., *Automorphism groups of supergraphs of the power graph of a finite group*, European J. Combin. **60** (2017), 82–88.
- [9] Mazurov V.D., Khukhro E.I., *Unsolved Problems in Group Theory*, Kourovka Notebook, Novosibirsk, Inst. Mat. Sibirsk. Otdel. Akad., 2006.
- [10] Shi W.-J., *A characterization of $U_3(2^n)$ by their element orders*, Xinan Shifan Daxue Xuebao Ziran Kexue Ban **25** (2000), no. 4, 353–360.
- [11] Ward H.N., *On Ree’s series of simple groups*, Trans. Amer. Math. Soc. **121** (1966), 62–89.
- [12] Weisner L., *On the number of elements of a group which have a power in a given conjugate set*, Bull. Amer. Math. Soc. **31** (1925), no. 9–10, 492–496.
- [13] Williams J.S., *Prime graph components of finite groups*, J. Algebra **69** (1981), no. 2, 487–513.
- [14] Wilson R.A., *The Finite Simple Groups*, Graduate Texts in Mathematics, 251, Springer, London, 2009.
- [15] Zhang Q., Shi W., Shen R., *Quasirecognition by prime graph of the simple groups $G_2(q)$ and ${}^2B_2(q)$* , J. Algebra Appl. **10** (2011), no. 2, 309–317.