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Iterated arc graphs

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Abstract: The arc graph $\delta(G)$ of a digraph G is the digraph with the set of arcs of G as vertex-set, where the arcs of $\delta(G)$ join consecutive arcs of G. In 1981, S. Poljak and V. Rödl characterized the chromatic number of $\delta(G)$ in terms of the chromatic number of G when G is symmetric (i.e., undirected). In contrast, directed graphs with equal chromatic numbers can have arc graphs with distinct chromatic numbers. Even though the arc graph of a symmetric graph is not symmetric, we show that the chromatic number of the iterated arc graph $\delta^k(G)$ still only depends on the chromatic number of G when G is symmetric. Our proof is a rediscovery of the proof of [Poljak S., *Coloring digraphs by iterated antichains*, Comment. Math. Univ. Carolin. **32** (1991), no. 2, 209–212], though various mistakes make the original proof unreadable.

Keywords: arc graph; chromatic number; free distributive lattice; Dedekind number
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