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Optimal control of a frictionless contact problem with normal compliance

Abstract: We consider a mathematical model which describes a contact between an elastic body and a foundation. The contact is frictionless with normal compliance. The goal of this paper is to study an optimal control problem which consists of leading the stress tensor as close as possible to a given target, by acting with a control on the boundary of the body. We state an optimal control problem which admits at least one solution. Next, we establish an optimality condition corresponding to a regularization of the model. We also introduce the regularized control problem for which we study the convergence when the regularization parameter tends to zero.

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