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On continuous self-maps and homeomorphisms of the Golomb space

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Abstract: The Golomb space $\mathbb{N}_\tau$ is the set $\mathbb{N}$ of positive integers endowed with the topology τ generated by the base consisting of arithmetic progressions $\{a + bn : n \geq 0\}$ with coprime a, b. We prove that the Golomb space $\mathbb{N}_\tau$ has continuum many continuous self-maps, contains a countable disjoint family of infinite closed connected subsets, the set Π of prime numbers is a dense metrizable subspace of $\mathbb{N}_\tau$, and each homeomorphism h of $\mathbb{N}_\tau$ has the following properties: $h(1) = 1$, $h(\Pi) = \Pi$, $\Pi_{h(x)} = h(\Pi_x)$, and $h(x^{\mathbb{N}}) = h(x)^{\mathbb{N}}$ for all $x \in \mathbb{N}$. Here $x^{\mathbb{N}} := \{x^n : n \in \mathbb{N}\}$ and Π_x denotes the set of prime divisors of x.

Keywords: Golomb space; arithmetic progression; superconnected space; homeomorphism
AMS Subject Classification: 54D05, 11A41

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