Alan Dow, Saharon Shelah

Asymmetric tie-points and almost clopen subsets of $\mathbb{N}^*$

Comment.Math.Univ.Carolin. 59,4 (2018) 451 –466.

Abstract: A tie-point of compact space is analogous to a cut-point: the complement of the point falls apart into two relatively clopen non-compact subsets. We review some of the many consistency results that have depended on the construction of tie-points of $\mathbb{N}^*$. One especially important application, due to Veličković, was to the existence of nontrivial involutions on $\mathbb{N}^*$. A tie-point of $\mathbb{N}^*$ has been called symmetric if it is the unique fixed point of an involution. We define the notion of an almost clopen set to be the closure of one of the proper relatively clopen subsets of the complement of a tie-point. We explore asymmetries of almost clopen subsets of $\mathbb{N}^*$ in the sense of how may an almost clopen set differ from its natural complementary almost clopen set.

Keywords: ultrafilter; cardinal invariants of continuum
AMS Subject Classification: 54D80, 03E15

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