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On maps preserving connectedness and/or compactness

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Abstract: We call a function $f\colon X \to Y$ P-preserving if, for every subspace $A \subset X$ with property P, its image $f(A)$ also has property P. Of course, all continuous maps are both compactness- and connectedness-preserving and the natural question about when the converse of this holds, i.e. under what conditions such a map is continuous, has a long history. Our main result is that any nontrivial product function, i.e. one having at least two nonconstant factors, that has connected domain, T_1 range, and is connectedness-preserving must actually be continuous. The analogous statement badly fails if we replace in it the occurrences of "connected" by "compact". We also present, however, several interesting results and examples concerning maps that are compactness-preserving and/or continuum-preserving.

Keywords: compactness; connectedness; preserving compactness; preserving connectedness

AMS Subject Classification: 54C05, 54D05, 54F05, 54B10

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