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The Golomb space is topologically rigid

Comment.Math.Univ.Carolin. 62,3 (2021) 347–360.

Abstract: The Golomb space \mathbb{N}_τ is the set \mathbb{N} of positive integers endowed with the topology τ generated by the base consisting of arithmetic progressions $\{a + bn : n \geq 0\}$ with coprime a, b . We prove that the Golomb space \mathbb{N}_τ is topologically rigid in the sense that its homeomorphism group is trivial. This resolves a problem posed by T. Banakh at Mathoverflow in 2017.

Keywords: Golomb topology; topologically rigid space

AMS Subject Classification: 11A99, 54G15

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