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Revisiting linear Weingarten spacelike submanifolds immersed in a locally symmetric semi-Riemannian space

Comment.Math.Univ.Carolin. 64,1 (2023) 39 –61.

Abstract: In this paper, we deal with n-dimensional complete linear Weingarten spacelike submanifolds immersed with parallel normalized mean curvature vector field and flat normal bundle in a locally symmetric semi-Riemannian space L_p^{n+p} of index $p > 1$, which obeys some curvature constraints (such an ambient space can be regarded as an extension of a semi-Riemannian space form). Under appropriate hypothesis, we are able to prove that such a spacelike submanifold is either totally umbilical or isometric to an isoparametric submanifold of the ambient space. For this, we use three main core analytical tools: a suitable version of the Omori–Yau maximum principle, parabolicity with respect to a modified Cheng–Yau operator and a certain integrability property.

Keywords: locally symmetric semi-Riemannian space; mean curvature vector field; complete linear Weingarten spacelike submanifold; totally umbilical submanifold; isoparametric submanifold; $\mathcal{L}$-parabolicity
AMS Subject Classification: 53C42, 53C21, 53C50

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