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More on exposed points and extremal points of convex sets in $\mathbb{R}^n$ and Hilbert space

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Abstract: Let $\mathbb{V}$ be a separable real Hilbert space, $k \in \mathbb{N}$ with $k < \dim \mathbb{V}$, and let B be convex and closed in $\mathbb{V}$. Let $\mathcal{P}$ be a collection of linear k-subspaces of $\mathbb{V}$. A point $w \in B$ is called exposed by $\mathcal{P}$ if there is a $P \in \mathcal{P}$ so that $(w + P) \cap B = \{w\}$. We show that, under some natural conditions, B can be reconstituted as the convex hull of the closure of all its exposed by $\mathcal{P}$ points whenever $\mathcal{P}$ is dense and G_δ. In addition, we discuss the question when the set of exposed by some $\mathcal{P}$ points forms a G_δ-set.

Keywords: convex set; extremal point; exposed point; Hilbert space; Grassmann manifold
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