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Universality, complexity and asymptotically uniformly smooth Banach spaces

Abstract: For $1 < p \leq \infty$, we show the existence of a Banach space which is both injectively and surjectively universal for the class of all separable Banach spaces with an equivalent p-asymptotically uniformly smooth norm. We prove that this class is analytic complete in the class of separable Banach spaces. These results extend previous works by N. J. Kalton, D. Werner and O. Kurka in the case $p = \infty$.

Keywords: asymptotic smoothness in Banach space; universality; complexity
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