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*Free locally convex spaces and  $L$ -retracts*

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**Abstract:** We study the relation of  $L$ -equivalence defined between Tychonoff spaces, that is, we study the topological isomorphisms of their respective free locally convex spaces. We introduce the concept of an  $L$ -retract in a Tychonoff space in terms of the existence of a special kind of simultaneous extensions of continuous functions, explore the relation of this concept with the Dugundji extension theorem, and find some conditions that allow us to identify  $L$ -retracts in various classes of topological spaces. As applications, we present a method for constructing examples of  $L$ -equivalent mappings and  $L$ -equivalent spaces and in particular, we show that the properties of being an open mapping or a perfect mapping are not  $L$ -invariant.

**Keywords:** free locally convex space;  $L$ -equivalence; retraction

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