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Special sets of reals and weak forms of normality on Isbell–Mrówka spaces

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Abstract: We recall some classical results relating normality and some natural weakenings of normality in Ψ -spaces over almost disjoint families of branches in the Cantor tree to special sets of reals like Q -sets, λ -sets and σ -sets. We introduce a new class of special sets of reals which corresponds to the corresponding almost disjoint family of branches being \aleph_0 -separated. This new class fits between λ -sets and perfectly meager sets. We also discuss conditions for an almost disjoint family \mathcal{A} being potentially almost-normal (pseudonormal), in the sense that \mathcal{A} is almost-normal (pseudonormal) in some c.c.c. forcing extension.

Keywords: Isbell–Mrówka spaces; almost disjoint families; almost-normal; weak λ -set

AMS Subject Classification: 54D15, 54D80

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