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A countably cellular topological group all of whose countable subsets are closed need not be $\mathbb{R}$-factorizable

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Abstract: We construct a Hausdorff topological group G such that $\aleph_1$ is a precalibre of G (hence, G has countable cellularity), all countable subsets of G are closed and C-embedded in G, but G is not $\mathbb{R}$-factorizable. This solves Problem 8.6.3 from the book "Topological Groups and Related Structures" (2008) in the negative.

Keywords: $\mathbb{R}$-factorizable; cellularity; C-embedded; Sorgenfrey line; P-group; Dieudonné completion; Hewitt–Nachbin completion; Bohr topology
AMS Subject Classification: 22A05, 54H11, 54D30, 54G20

References

[1] Arhangel'skiĭ A. V., Tkachenko M. G., *Topological Groups and Related Structures*, Atlantis Stud. Math., 1, Atlantis Press, Paris World Scientific Publishing Co., Hackensack, 2008.
[2] Engelking R., *General Topology*, Sigma Ser. Pure Math., 6, Heldermann, Berlin, 1989.
[3] Noble N., Ulmer M., *Factoring functions on Cartesian products*, Trans. Amer. Math. Soc. **163** (1972), 329–339.
[4] Reznichenko E. A., Sipacheva O. V., *The free topological group on the Sorgenfrey line is not $\mathbb{R}$-factorizable*, Topology Appl. **160** (2013), no. 11, 1184–1187.
[5] Sipacheva O. V., *Free Boolean topological groups*, Axioms **4** (2015), no. 4, 492–517.
[6] Tkachenko M. G., *Some results on inverse spectra. II*, Comment. Math. Univ. Carolin. **22** (1981), no. 4, 819–841.
[7] Tkachenko M. G., *Cellularity in quotient spaces of topological groups*, Forum Math. **31** (2019), no. 2, 351–359.
[8] Xie L.-H., Yan P.-F., *The continuous d-open homomorphism images and subgroups of $\mathbb{R}$-factorizable paratopological groups*, Topology Appl. **300** (2021), Paper No. 107627, 7 pages.