Eva Fašangová Asymptotic analysis for a nonlinear parabolic equation on $\mathbb R$

Comment.Math.Univ.Carolinae 39,3 (1998) 525-544.

Abstract: We show that nonnegative solutions of

$$u_t - u_{xx} + f(u) = 0, \quad x \in \mathbb{R}, \quad t > 0,$$

$$u = \alpha \bar{u}, \quad x \in \mathbb{R}, \quad t = 0, \quad \text{supp } \bar{u} \text{ compact}$$

either converge to zero, blow up in L²-norm, or converge to the ground state when $t \to \infty$, where the latter case is a threshold phenomenon when $\alpha > 0$ varies. The proof is based on the fact that any bounded trajectory converges to a stationary solution. The function f is typically nonlinear but has a sublinear growth at infinity. We also show that for superlinear f it can happen that solutions converge to zero for any $\alpha > 0$, provided supp \bar{u} is sufficiently small.

Keywords: parabolic equation, stationary solution, convergence **AMS Subject Classification:** 35B40, 35K55, 35B05