An interesting class of ideals in subalgebras of $C(X)$ containing $C^*(X)$

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Abstract. In the present paper we give a duality between a special type of ideals of subalgebras of $C(X)$ containing $C^*(X)$ and z-filters of βX by generalization of the notion z-ideal of $C(X)$. We also use it to establish some intersecting properties of prime ideals lying between $C^*(X)$ and $C(X)$. For instance we may mention that such an ideal becomes prime if and only if it contains a prime ideal. Another interesting one is that for such an ideal the residue class ring is totally ordered if and only if it is prime.

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1. Introduction

Throughout the paper all topological spaces are assumed to be Tychonoff. For a space X, $C(X)$ stands for the ring of all real valued continuous functions on X, $C^*(X)$ is the subring of $C(X)$ consisting of all bounded functions and $\Sigma(X)$ will denote the collection of all subalgebras of $C(X)$ containing $C^*(X)$.

It is a fascinating fact in the theory of rings of continuous functions that for a space X the structure spaces of both $C(X)$ and $C^*(X)$ produce the Stone-Čech compactification βX of that space. Plank [7] has proved that the structure space of any subalgebra of $C(X)$ containing $C^*(X)$ also produces the Stone-Čech compactification βX of X in an analogous manner. In this course an analogous study of arbitrary subalgebra of $C(X)$ containing $C^*(X)$ becomes important. The study of ideals in $C(X)$ depends strongly on the fact that if I is a proper ideal in $C(X)$ then $Z(I) = \{Z(f) : f \in I\}$ becomes a z-filter on X. But in case of an arbitrary $A(X) \in \Sigma(X)$ the analogous statement is not necessarily true. H.L. Byun and S. Watson [2] introduced a method for studying ideals in arbitrary $A(X) \in \Sigma(X)$. For each ideal I in $A(X)$, they associated a family of subsets of X given by $\mathcal{Z}_A[I] = \bigcup\{\mathcal{Z}_A(f) : f \in I\}$, where for each $f \in A(X)$, $\mathcal{Z}_A(f) = \{E \in Z(X) : \exists\, g \in A(X) \text{ with } f \cdot g|_{X-E} = 1\}$, which latter turned out to be a z-filter on X. Further they called an ideal I in $A(X)$ a $\mathcal{B}$-ideal if $\mathcal{Z}_A^{-1}[\mathcal{Z}_A[I]] = I$. But the map $\mathcal{Z}_A$, which relates ideals in $A(X)$ to z-filters on X, lacks the sensitivity for distinguishing prime ideals. In fact even in case of

$A(X) = C(X)$ also, it follows that $\mathcal{Z}_C[O_C^p] = \mathcal{Z}_C[M_C^p]$ for all $p \in \beta X$, where $O_C^p = \{f \in C(X) : p \in \operatorname{int}_{\beta X}\{\operatorname{cl}_{\beta X} Z(f)\}\}$. More generally, if P is a prime ideal contained in a maximal ideal M_A^p in $A(X)$ then $\mathcal{Z}_A[P] = \mathcal{Z}_A[M_A^p]$. So by this definition of $\mathcal{B}$-ideal there does not exist any non-maximal prime $\mathcal{B}$-ideal. In this article we introduce a new type of ideals in $A(X)$ called z_A^β-*ideals*, and a correspondence z_A^β from the set of all ideals in $A(X)$ to the set of a special type of filters in βX in such a way that the correspondence z_A^β retains the sensitivity of distinguishing prime ideals to some extent. In fact we shall show that there exists a non-maximal prime z_A^β-ideal in $A(X)$. Following Plank [7], for any $f \in A(X)$ we denote $\{p \in \beta X : (f \cdot g)^*(p) = 0$ for all $g \in A(X)\}$ as $S_A(f)$ and $Z_A^\beta[I] = \{S_A(f) : f \in I\}$. Throughout this article we shall call $S_A(f)$ an A-*zeroset* in βX, and the set $\{S_A(f) : f \in A(X)\}$ will be denoted by $Z_A^\beta[X]$.

2. z_A^β-filter on βX

Like z-filters in X, we define z_A^β-filters in βX in the following way.

Definition 2.1. A non empty subset $\mathcal{F}$ of $Z_A^\beta[X]$ is called a z_A^β-*filter* on βX provided that

(1) $\varphi \notin \mathcal{F}$,
(2) if Z_1, Z_2 are in $\mathcal{F}$ then $Z_1 \cap Z_2 \in \mathcal{F}$,
(3) if Z is in $\mathcal{F}$ and $Z' \in Z_A^\beta[X]$ with $Z' \supset Z$ then $Z' \in \mathcal{F}$.

Now we can easily see that if f is a unit of $A(X)$ then $\frac{1}{f} \in A(X)$ so that $(f \cdot \frac{1}{f})^*(p) = 1$ for all $p \in \beta X$ and therefore $S_A(f) = \varphi$. Again for each $p \in \beta X$ there exists $g_p \in A(X)$ such that $(f \cdot g_p)^*(p) \neq 0$. This means that f is missed by every maximal ideal in $A(X)$, so that f is not a unit of $A(X)$. Therefore we have the following lemma.

Lemma 2.2. *Suppose $A(X) \in \Sigma(X)$. Then for any $f \in A(X)$, $S_A(f) = \varphi$ if and only if f is a unit of $A(X)$.*

The above lemma discovers the duality existing between the ideals of $A(X)$ and z_A^β-filters on βX.

Theorem 2.3. *For any $A(X) \in \Sigma(X)$ the following holds.*

(1) *If I is an ideal in $A(X)$ then the family $Z_A^\beta[I] = \{S_A(f) : f \in I\}$ is a z_A^β-filter on βX.*
(2) *If $\mathcal{F}$ is a z_A^β-filter on βX then the family $Z_A^{\beta-1}[\mathcal{F}]$ given as $\{f \in A(X) : S_A(f) \in \mathcal{F}\}$ is an ideal in $A(X)$.*

Before talking about the duality between maximal ideals in $A(X)$ and maximal z^β_A-filter in βX we simply write down the following results, whose proofs can also be given by using the well-known routine arguments. First we introduce the following notion.

Definition 2.4. A z^β_A-*ultrafilter* on βX is a z^β_A-filter on βX which is not contained in any other z^β_A-filter on βX.

Theorem 2.5. *For any $A(X) \in \Sigma(X)$ the followings are equivalent.*

(1) *Every z^β_A-filter on βX can be extended to a z^β_A-ultrafilter on βX.*

(2) *Every subfamily of $Z^\beta_A[X]$ with finite intersection property can be extended to a z^β_A-ultrafilter on βX and therefore a z^β_A-ultrafilter on βX is a subfamily of $Z^\beta_A[X]$ which is maximal with respect to having finite intersection property. Conversely a subfamily $\mathcal{F}$ of $Z^\beta_A[X]$ enjoying finite intersection property and maximal with respect to this property is necessary a z^β_A-ultrafilter on βX.*

(3) *A z^β_A-filter $\mathcal{F}$ on βX is a z^β_A-ultrafilter on βX if and only if for any $Z \in Z^\beta_A[X]$, $Z \cap Z' \neq \varphi$ for any $Z' \in \mathcal{F}$, implies that $Z \in \mathcal{F}$.*

As a straightforward consequence of the above theorem, taking into account the maximality of M and $\mathcal{F}$, we have the following theorem.

Theorem 2.6. *Suppose $A(X) \in \Sigma(X)$. Then*

(1) *if M is a maximal ideal in $A(X)$ then $Z^\beta_A[M]$ is a z^β_A-ultrafilter on βX,*

(2) *if $\Im$ is a z^β_A-ultrafilter on βX then $Z^{\beta-1}_A[\Im]$ is a maximal ideal in $A(X)$.*

Using the duality between maximal ideals in $A(X)$ and ultrafilters in βX we have the following theorem.

Theorem 2.7. *Let $A(X) \in \Sigma(X)$ and $f \in A(X)$. If M is a maximal ideal in $A(X)$ and $S_A(f)$ meets every member of $Z^\beta_A[M]$ then $f \in M$.*

3. z^β_A-ideals in $A(X)$ and its properties

For any $A(X) \in \Sigma(X)$ and for any z^β_A-filter $\Im$ on βX, it is obvious that $\Im = Z^\beta_A[Z^{\beta-1}_A[\Im]]$; therefore Z^β_A can be considered to be a mapping from the set of all ideals in $A(X)$ onto the set of all z^β_A-filters on βX. Furthermore, for any ideal I in $A(X)$, we have $I \subset Z^{\beta-1}_A[Z^\beta_A[I]]$. The inclusion in the above relation may be proper. In fact in the ring $C(\mathbb{R})$ if we consider the ideal $I = \langle i \rangle$, the smallest ideal in $C(\mathbb{R})$ generated by the identity mapping i, we can easily observe that the mapping $i^{1/3}$ is in $Z^{\beta-1}_C[Z^\beta_C[I]]$ but it does not belong to I. This motivates to introduce the following definition.

Definition 3.1. An ideal I in $A(X) \in \Sigma(X)$ is said to be a z_A^β*-ideal* if for any $f \in A(X)$, $S_A(f) \in Z_A^\beta[I]$ implies that $f \in I$, that is, $I = Z_A^{\beta-1}[Z_A^\beta[I]]$.

Clearly if $\mathcal{F}$ is a z_A^β-filter on βX then $I = Z_A^{\beta-1}[\Im]$ is a z_A^β-ideal in $A(X)$, in fact $\Im = Z_A^\beta[Z_A^{\beta-1}[\Im]]$. Further for any $p \in \beta X$, $O_A^p = \{f \in A(X) : p \in \operatorname{int}_{\beta X} S_A(f)\}$ is a z_A^β-ideal. It is also evident that the intersection of any nonempty collection of z_A^β-ideals in $A(X)$ is again a z_A^β-ideal. Again from Theorem 2.7 we can prove that for any maximal ideal M in $A(X)$, $M = Z_A^{\beta-1}[Z_A^\beta[M]]$. Thus we have the following theorem.

Theorem 3.2. *Suppose $A(X) \in \Sigma(X)$. Then every maximal ideal in $A(X)$ is a z_A^β-ideal in $A(X)$.*

The following theorem shows that like maximal prime ideals, i.e. maximal ideals, minimal prime ideals in $A(X)$ are also z_A^β-ideals.

Theorem 3.3. *If I is a z_A^β-ideal in $A(X)$ and P is minimal in the class of prime ideals containing I, then P is a z_A^β-ideal.*

PROOF: Let J be a prime ideal containing I which is not a z_A^β-ideal. Then to prove the theorem it is sufficient to show that J is not minimal in the class of prime ideals containing I. Since J is not a z_A^β-ideal there exists an $f \in J$ and a $g \in A(X)$ with $g \notin J$ such that $S_A(f) = S_A(g)$. Now consider the set $S = (A(X) - J) \cup \{hf^n : h \notin J, n \in \mathbb{N}\}$. Since J is a prime ideal, S is closed under multiplication. Furthermore S does not meet I. In fact $hf^n \in I$ for some $h \in J$, $n \in \mathbb{N}$ implies that $h \cdot g \in J$, which contradicts that J is a prime ideal. Hence there exists a prime ideal containing I and disjoint from S and, hence, contained in J properly. Therefore J is not minimal. $\square$

Remark 3.4. Since the ideal $\langle 0 \rangle$ in any $A(X)$ is a z_A^β-ideal, every minimal prime ideal in an arbitrary $A(X)$ is a z_A^β-ideal.

It is well known that every z-ideal in $C(X)$ is the intersection of all prime ideals containing it. The basic fact behind the result is that $Z(f^n) = Z(f)$ for all $n \in \mathbb{N}$. In our setting of $A(X)$ we also see that $S_A(f^n) = S_A(f)$ for all $n \in \mathbb{N}$ and therefore we get the following theorem.

Theorem 3.5. *Every z_A^β-ideal in $A(X)$ is the intersection of all prime ideals in $A(X)$ containing it.*

Remark 3.6. Using Theorem 3.3 and Theorem 3.5 it is easy to observe that every z_A^β-ideal in $A(X)$ is the intersection of all minimal prime ideals containing it.

The following theorem shows that z_A^β-ideals in $A(X)$ are actually A-analogues of z-ideals in $C(X)$.

Theorem 3.7. *In $C(X)$, an ideal I is a z-ideal if and only if it is a z^β_C-ideal.*

Proof: Let I be a z-ideal in $C(X)$ and $f \in C(X)$ be such that $S_C(f) \in Z^\beta_C[I]$. Then there exists $g \in I$ such that $S_C(f) = S_C(g)$. Since it is well known that for any $f \in C(X)$, $S_C(f) = \mathrm{cl}_{\beta X} Z(f)$ and $\mathrm{cl}_{\beta X} Z(f) \bigcap X = Z(f)$, the above relation implies that $Z(f) = Z(g) \in Z[I]$. Hence $f \in I$, as I is a z-ideal. Therefore every z-ideal in $C(X)$ is also a z^β_C-ideal.

Conversely, let I be a z^β_C-ideal in $C(X)$ and $f \in C(X)$ with $Z(f) \in Z[I]$. Then there exists an element g of I such that $Z(f) = Z(g)$, so that $\mathrm{cl}_{\beta X} Z(f) = \mathrm{cl}_{\beta X} Z(g) \in Z^\beta_C[I]$. Since I is a z^β_C-ideal, it follows that $f \in I$, proving that I is a z-ideal in $C(X)$. □

It is known that in case of $C(X)$, an intersection of prime ideals need not be a z-ideal, see Example 2G.1 of [5]. So Theorem 3.7 shows that the converse of Theorem 3.5 is not valid. But like z-ideals in $C(X)$, a z^β_A-ideal in an arbitrary $A(X) \in \Sigma(X)$ can also be described as a purely algebraic object.

Theorem 3.8. *An ideal I in $A(X) \in \Sigma(X)$ is a z^β_A-ideal if and only if given $f \in A(X)$ there exists $g \in I$ such that whenever f belongs to every maximal ideal in $A(X)$ containing g, then $f \in I$.*

Proof: Let I be a z^β_A-ideal in $A(X)$ and $f \in A(X)$. Again let $g \in I$ be such that f belongs to every maximal ideal in $A(X)$ containing g. Then $S_A(g) \subset S_A(f)$ so that $S_A(f) \in Z^\beta_A[I]$. Since I is a z^β_A-ideal in $A(X)$, we have $f \in I$.

For the converse, let us assume that the given condition holds and $S_A(f) \in Z^\beta_A[I]$ for some $f \in A(X)$. Taking $f = g$ we see that f belongs to every maximal ideal in $A(X)$ that contains g. Hence $f \in I$ so that I is a z^β_A-ideal. □

Now we present an example which shows that the notion of $\mathcal{B}$-ideal in $A(X)$ [2], already described in Introduction, does not coincide with the notion of z^β_A-ideal even with the choice $A(X) = C(X)$.

Example. Let us consider the z-ideal $O_0 = \{f \in C(X) : 0 \in \mathrm{int}_X Z(f)\}$. Then the z-filter $\mathcal{Z}_C(i) = \{Z \in Z(\mathbb{R}) : \exists g \in C(\mathbb{R}) \text{ with } i \cdot g|_{\mathbb{R}-Z} = 1\} \subset \mathcal{Z}_C[O_0]$. In fact if $Z \in \mathcal{Z}_C(i)$ then there exists $g \in C(\mathbb{R})$ such that $i \cdot g|_{\mathbb{R}-Z} = 1$, which implies that $i \cdot g(\mathrm{cl}_{\mathbb{R}}(\mathbb{R} - Z)) = \{1\}$. It then clearly follows that $0 \notin \mathrm{cl}_{\mathbb{R}}(\mathbb{R} - Z)$. Therefore there exists a $\delta > 0$ such that $(\mathbb{R} - Z) \cap (-\delta, \delta) = \emptyset$. We define $h \in C(\mathbb{R})$ as follows: if $|x| \le \frac{\delta}{2}$ then $h(x) = 0$, if $\frac{\delta}{2} \le x \le \delta$ then $h(x) = \frac{g(\delta)}{\delta}(2x - \delta)$, if $|x| \ge \delta$ then $h(x) = g(x)$, and if $-\delta \le x \le -\frac{\delta}{2}$ then $h(x) = \frac{g(-\delta)}{-\delta}(2x + \delta)$.
Then clearly $h \in O_0$ and $i \cdot h|_{\mathbb{R}-Z} = 1$, so that $Z \in \mathcal{Z}_C(h)$. Hence $Z \in \mathcal{Z}_C[O_0]$. But as $i \notin O_0$, O_0 cannot be an $\mathcal{B}$-ideal in $C(\mathbb{R})$.

Next we recall the definition of e-ideal [5]. An ideal I in $C^*(X)$ is called an *e-ideal* if $E_\epsilon(f) \in E(I) = \bigcup_\epsilon E_\epsilon(f)$ for all $\epsilon > 0$ implies that $f \in I$, where

$E_\epsilon(f) = f^{-1}[(-\epsilon, \epsilon)]$. But the following example shows that the notion of e-ideal in $C^*(X)$ does not coincide with the notion of $z^\beta_{C^*}$-ideal.

Example. In the ring $C^*(\mathbb{R})$ let us consider the ideal $O_0 = \{f \in C^*(\mathbb{R}) : 0 \in \operatorname{int}_{\beta\mathbb{R}} Z(f^\beta)\}$. Since $Z(f^\beta) = S_{C^*}(f)$ for any $f \in C^*(\mathbb{R})$, it is easy to see that O_0 is a $z^\beta_{C^*}$-ideal in $C^*(\mathbb{R})$. Now taking $f = (i \vee -1) \wedge 1$ we see that $E_\epsilon(f) \in E(O_0)$ for all $\epsilon > 0$, but $f \notin O_0$. Hence O_0 is not an e-ideal.

In case of $C(X)$ it is well known that a z-ideal need not be prime. In fact if X is not an F-space then there exists some $p \in \beta X$ such that O^p_C is not a prime ideal. But O^p_C is a z-ideal for every $p \in \beta X$, i.e. a z^β_C-ideal. The following theorem tells us that if a z^β_A-ideal contains a prime ideal then it becomes prime.

Theorem 3.9. *Suppose $A(X) \in \Sigma(X)$ and let I be a z^β_A-ideal in $A(X)$. Then the following statements are equivalent.*

(1) *I is a prime ideal in $A(X)$.*
(2) *I contains a prime ideal in $A(X)$.*
(3) *For all g, h in $A(X)$, $g \cdot h = 0$ implies that $g \in I$ or $h \in I$.*
(4) *For every $f \in A(X)$ there exists an A-zero set Z in $Z^\beta_A[I]$ such that either*

$$M^p_A(f) \geq 0 \ \forall p \in Z \quad \text{or} \quad M^p_A(f) \leq 0 \ \forall p \in Z.$$

Proof: (1) $\Rightarrow$ (2) is trivial.

(2) $\Rightarrow$ (3) Let us assume that P is a prime ideal in $A(X)$ contained in I. Now for any two g, h in $A(X)$ if $g \cdot h = 0$ then $g \cdot h \in P$. So either $g \in P$ or $h \in P$, that is, either $g \in I$ or $h \in I$.

(3) $\Rightarrow$ (4) For any given $f \in A(X)$, $(f \vee 0) \cdot (f \wedge 0) = 0$. Hence from (3) it follows that $f \vee 0 \in I$ or $f \wedge 0 \in I$. If $f \vee 0 \in I$ then $S_A(f \vee 0) \in Z^\beta_A[I]$. In this case for any $p \in S_A(f \vee 0)$, we have $f \vee 0 \in M^p_A$, that is, $M^p_A(f) \vee 0 = 0$. Clearly this implies that $M^p_A(f) \leq 0$ for all $p \in S_A(f \vee 0) \in Z^\beta_A[I]$. Similarly in case $f \wedge 0 \in I$ we have $M^p_A(f) \geq 0$ for all $p \in S_A(f \wedge 0) \in Z^\beta_A[I]$.

(4) $\Rightarrow$ (1) Let us assume $g \cdot h \in I$, $g, h \in A(X)$, and consider the function $|g| - |h|$ in $A(X)$. Then there exists an A-zeroset Z such that $M^p_A(|g| - |h|) \geq 0$ for all $p \in Z$, say for definiteness. Then clearly

$$M^p_A(|g|) \geq [M^p_A(|h|) \ \text{ for all } \ p \in Z.$$

Now we claim that $Z \cap S_A(g \cdot h) = Z \cap S_A(h) \subset S_A(h)$. In fact, by the above relation, $p \in S_A(g) \cap Z$ implies that $p \in S_A(h) \cap Z$, here we use the absolute convexity of maximal ideals in $A(X)$. Now because $S_A(f \cdot g) \in Z^\beta_A[I]$, it follows that $S_A(h) \in Z^\beta_A[I]$. Therefore I is a z^β_A-ideal and we have $h \in I$. Analogously, if

$M^p_A(|g| - |h|) \geq 0$ for all $p \in Z$, then we would have obtained $g \in I$. Hence I is a prime ideal in $A(X)$. $\square$

In [6] we have observed that in any uniformly closed ϕ-*algebra* every prime ideal can be extended to a unique maximal ideal, where by a ϕ-*algebra* we mean an archimedean lattice ordered algebra over the real field $\mathbb{R}$ which has an identity element 1 that is a weak order unit (i.e. $x \wedge 0$ implies $x = 0$) and it is called *uniformly closed* if every Cauchy sequence of its elements converges in it. Here we present a different proof of the above result for arbitrary $A(X) \in \Sigma(X)$. We recall that in any commutative ring if I and J are two prime ideals neither containing the other then $I \cap J$ is not a prime ideal. Therefore in arbitrary $A(X) \in \Sigma(X)$ if two distinct maximal ideals contain a single prime ideal we get a contradiction as intersection of two maximal ideals is a z^β_A-ideal in $A(X)$ and by the above theorem any z^β_A-ideal containing a prime ideal is prime. This gives an alternative proof of the following theorem.

Theorem 3.10. *Every prime ideal in an $A(X) \in \Sigma(X)$ can be extended to a unique maximal ideal.*

To end this article we are interested in knowing when a partially ordered residue class ring modulo a z^β_A-ideal is *totally ordered*. The following theorem shows that these are only when z^β_A-ideals are prime. We recall that every prime ideal in arbitrary $A(X) \in \Sigma(X)$ is absolutely convex. From this it is easy to conclude that every z^β_A-ideal is also absolutely convex.

Theorem 3.11. *Suppose that $A(X) \in \Sigma(X)$ and that I is a z^β_A-ideal in $A(X)$. Then $A(X)/I$ is totally ordered if and only if I is prime.*

Proof: Let $A(X)/I$ be a totally ordered ring and $f \in A(X)$. We assume that $I(f) \geq 0$. Since I is absolutely convex we have $f - |f| \in I$, and therefore $S_A(f) \in Z^\beta_A[I]$. Hence for any $p \in S_A(f)$ it follows that $M^p_A(f - |f|) = 0$ that is $M^p_A(f) = M^p_A(|f|)$. This implies that $M^p_A(f) \geq 0$ for all $p \in Z = S_A(f - |f|) \in Z^\beta_A[I]$. Therefore by Theorem 3.9 I becomes a prime ideal.

Conversely let I be a prime ideal in $A(X)$ and $f \in A(X)$. Then again by Theorem 3.9 there exists a $Z \in Z^\beta_A[I]$ such that either $M^p_A(f) \geq 0$ for all $p \in Z$ or $M^p_A(f) \leq 0$ for all $p \in Z$. Let us assume that $M^p_A(f) \geq 0$ for all $p \in Z$. This implies that $f - |f| \in M^p_A$ so that $M^p_A(f) = M^p_A(|f|)$ for all $p \in Z$. Hence $M^p_A(f - |f|) = 0$ for all $p \in Z$, that is $Z \subset S_A(f - |f|)$. Now as $Z^\beta_A[I]$ is a z^β_A-filter on βX and I is a z^β_A-ideal in $A(X)$ we have $f - |f| \in I$ and hence $I(f) \geq 0$. Similarly $M^p_A(f) \leq 0$ for all $p \in Z$ implies that $I(f) \leq 0$. Therefore $A(X)/I$ becomes totally ordered. $\square$

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