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Seeable matter; unseeable antimatter

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The universe we see gives every sign of being composed of matter. This is considered a major unsolved problem in theoretical physics. Using the mathematical modeling based on the algebra $\mathbf{T} := \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$, an interpretation is developed that suggests that this seeable universe is not the whole universe; there is an unseeable part of the universe composed of antimatter galaxies and stuff, and an extra 6 dimensions of space (also unseeable) linking the matter side to the antimatter — at the very least.

Bases for the real division algebras, $\mathbf{C}$, $\mathbf{H}$, $\mathbf{O}$ (complex algebra, quaternions, and octonions), are [1], [2], [7]:

$$\begin{array}{ll} \mathbf{C} & \{1,\ i\} \\ \mathbf{H} & \{q_0 = 1,\ q_k,\ k = 1,2,3\} \\ \mathbf{O} & \{e_0 = 1,\ e_a,\ a = 1,\dots,7\} \end{array}$$

The algebra

$$\mathbf{T} = \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$$

is $2 \times 4 \times 8 = 64$-dimensional. It is noncommutative, nonassociative, and nonalternative.

Although I consider it but a restricted model of reality, the basis of what I will do here is the 10-dimensional space-time model developed in [1] (Chapters 2 to 6), with mathematical expansion to be found in [2] (Chapters 2, 3 and 11). In this model, which accounts for a single family of quarks and leptons, and a corresponding antifamily, the foundation is the 128-dimensional hyperspinor space

$$\mathbf{T}^2$$

(the doubling of $\mathbf{T}$ in the spinor space is modeled on the notion that a Dirac spinor is a double Pauli spinor).

A Dirac spinor is acted upon by the Dirac algebra,

$$\mathbf{C}(4) \simeq \mathbf{P}(2),$$

where the Pauli algebra

$$\mathbf{P} \simeq \mathbf{C}(2) \simeq \mathbf{C} \otimes \mathbf{H}.$$

This is the complexification of the Clifford algebra of 1,3-spacetime. Likewise $\mathbf{T}^2$ is acted upon by the complexification of the Clifford algebra of 1,9-spacetime, represented by

$$\mathbf{T}_L(2),$$

where $\mathbf{T}_L$ is the algebra of left actions of $\mathbf{T}$ on itself, which in the octonion case, due to nonassociativity, requires the nesting of actions (see, for example, [1, Chapter 2] and [2, Section 2.4]; and for more background material, [3], [4], and [5] (the work of Gürsey at Yale University during the 1970s was the inspiration for all of my work — and that of many others — applying the octonion algebra to physics)).

The work of Gürsey (and Günaydin) was inspired by the work of von Neumann, Jordan and Wigner [8], who investigated an expansion of quantum theory from a foundation on $\mathbf{C}$ to one on $\mathbf{O}$. They linked quantum observability with algebraic associativity, and unobservability with nonassociativity, thinking along these lines being forced by the nonassociativity of $\mathbf{O}$. (I do not know the details of their work, but the notion that nonassociativity could be associated with things unseen, and unseeable, partly motivated this work.)

The quantum notion of unobservable is more restrictive than the notion of unseeable being used here. In particular, quarks are unseeable, but they are detectable, and they supply the paradigm — albeit not well defined — of what is meant by unseeable. But being not well defined is not a problem. My working assumption is that this model is but a kernel of something much larger, much deeper, and, I hope, ultimately knowable. It may not be.

The model building in [1], [2] relies heavily on a resolution of the identity of

$$\mathbf{S} := \mathbf{C} \otimes \mathbf{O}$$

into a pair of orthogonal idempotents,

$$\rho_\pm = \frac{1}{2}(1 \pm ie_7).$$

These satisfy

$$\rho_\pm e_p \rho_\pm = e_p \rho_\mp \rho_\pm = 0,\ p = 1,2,3,4,5,6,$$

and

$$\rho_\pm e_k \rho_\pm = e_k \rho_\pm \rho_\pm = e_k \rho_\pm,\ k = 0,7$$

(nonassociativity does not play a role here, so no parentheses are required; also note that $e_7\rho_\pm = \mp i\rho_\pm$). (This is the same resolution exploited by Gürsey, et al. [5], and numerous other places in the years following. As is done here, it is how they gave rise to the $SU(3)$ color group out of the octonion automorphism group, G_2 (see [1, Chapter 2]).) With these projectors $\mathbf{S}$ can be divided into 4 orthogonal subspaces:

$$\begin{aligned} \mathbf{S}_{++} &= \rho_+ \mathbf{S} \rho_+, \\ \mathbf{S}_{--} &= \rho_- \mathbf{S} \rho_-, \\ \mathbf{S}_{+-} &= \rho_+ \mathbf{S} \rho_-, \\ \mathbf{S}_{-+} &= \rho_- \mathbf{S} \rho_+. \end{aligned}$$

Both $\mathbf{S}_{++}$ and $\mathbf{S}_{--}$ are associative subalgebras of $\mathbf{S}$ isomorphic to $\mathbf{C}$. $\mathbf{S}_{+-}$ and $\mathbf{S}_{-+}$ are not subalgebras, and they are highly nonassociative (this nonassociativity implying $\mathbf{S}_{\pm\mp}^2 = \mathbf{S}_{\mp\pm}$ (you'd better check that — it is not relevant, but I never noticed that before — hmm)). Anyway, elements of the first two sets are linear (over $\mathbf{C}$) in the octonions $\{e_0 = 1,\ e_7\}$, and the second two sets linear over $\{e_p,\ p = 1,2,3,4,5,6\}$.

With respect to the $SU(3)$ subgroup of the octonion automorphism group G_2 that leaves the unit e_7 fixed these parts of $\mathbf{S}$ transform, respectively, as a singlet, anti-singlet, triplet, and anti-triplet. That is,

$$\begin{aligned} &\rho_+ \mathbf{S} \text{ is matter;} \\ &\rho_- \mathbf{S} \text{ is anti-matter.} \end{aligned}$$

The same is true if we replace $\mathbf{S}$ by $\mathbf{T}^2$.

An elegant representation of the Clifford algebra $\mathcal{CL}(1,9)$ represented in $\mathbf{T}_L(2)$, that is aligned with the choice of the octonion unit e_7 to appear in $\rho_\pm$, arises from the following set of ten anti-commuting 1-vectors:

$$\beta,\ \ \gamma q_{Lk} e_{L7},\ k = 1,2,3,\ \ \gamma i e_{Lp},\ p = 1,...,6,$$

where

$$\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \alpha = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \gamma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

and as usual the subscripts "L" and "R" signify an action from the left or the right on $\mathbf{T}$. (So, for example,

$$\mathbf{S}_{+-} = \rho_+ \mathbf{S} \rho_- = \rho_{L+} \rho_{R-} [\mathbf{S}].)$$

(Note: $\mathcal{CL}(1,9)$ can be represented in other ways, and certainly using only complex matrices. A principal underlying this work is that the division algebras should be as generative in physics as they are in mathematics, and that, with the Dirac algebra and its spinors as a guide, this model of 1,9-spacetime, with its spinor space consisting of a family and antifamily of quark and lepton Dirac spinors, falls out relatively naturally, if one pays attention to the structure of the underlying mathematics. The alignment of this representation of $\mathcal{CL}(1,9)$ with the ρ projectors is not necessary, but then neither is a combination necessary to open a safe. Dynamite will do. By ordering things as I have, I am attempting to demonstrate the elegant way the mathematics elucidates the physics — to provide a combination with which the goodies in this safe can be more easily grasped, and the essential nature of the mathematics made more clear.)

Here is the working assumption upon which this work is based: if we project out from this model those bits we know are unseeable (anything carrying a color charge), what is left will be seeable, and everything that is gone will be unseeable (even if it does not carry a color charge). As it stands, this is the model of a universe with 10 dimensions, containing both matter and anti-matter in the form of leptons and quarks, and their anti-particles. The quarks and the extra 6 space dimensions are unseeable. There exist models that attempt to explain quark confinement, but so long as they remain confined we are safe in labeling them unseeable.

Quarks carry $SU(3)$ color charges, as do the extra 6 spaces dimensions in this model. Both are unseeable (this is both an assumption, and an observation), and both can be projected out of the model in the same way. Since the color charges reside in the octonion units e_p, $p = 1, \dots, 6$, we need merely use the $\rho_\pm$ to get rid of them.

Start with the 6 extra space dimensions. There are two (what I would call) canonical ways of reducing the 1-vectors of $\mathcal{CL}(1,9)$, a mix of seeable and unseeable dimensions, to the 1-vectors of seeable $\mathcal{CL}(1,3)$ (that is, we are using the ρ projectors to eliminate bits that carry the unseeable color charge, here the extra 6 space dimensions):

$$\rho_{L\pm}\ \{\beta,\ \ \gamma q_{Lk} e_{L7},\ k = 1,2,3,\ \ \gamma i e_{Lp},\ p = 1, ..., 6\}\rho_{L\pm}$$

$$=\ \{\beta,\ \ \gamma i q_{Lk},\ k = 1,2,3\}\rho_{L\pm}.$$

These two collections of $\mathcal{CL}(1,3)$ 1-vectors act on half of the full spinor space $\mathbf{T}^2$. In particular, they act respectively on

$$\rho_{L\pm}[\mathbf{T}^2] = \rho_\pm \mathbf{T}^2,$$

where the underlying mathematics implies that these are, respectively, the matter and anti-matter halves of $\mathbf{T}^2$ ($\rho_+\mathbf{T}^2$ being a full family of lepton and quark Dirac spinors, and $\rho_-\mathbf{T}^2$ the corresponding anti-family: see [1, Chapters 3 and 4], and [2, Section 3.2]).

And this is the point: once 1,9-spacetime is reduced to 1,3-spacetime (the unseeable part projected away), one discovers that half of the hyper-spinor space is also projected away, and it too — given the interpretation of the mathematics we are adopting here — should be unseeable, even though bits of it do not carry the color charge (anti-leptons). That is, from the 1,3-spacetime that is left you can see only the matter half of $\mathbf{T}^2$, or the antimatter half. One or the other is projected away, along with things carrying the color charge, and so this antimatter universe should too be unseeable. We think of our universe as being composed of matter (stars, planets, and such; the production of individual antimatter particles is not considered a problem). The antimatter half of $\mathbf{T}^2$ is not gone, nor are the extra 6 space dimensions. We just do not directly see them.

Quarks, like the extra 6 dimensions of space in this model, are also unseeable. And like the extra 6 dimensions of space, they owe their existence to the octonion units $e_p,\ p = 1, 2, 3, 4, 5, 6$. To reduce the spinor space $\mathbf{T}^2$ all the way to its observable lepton part (the anti-lepton part is similar) we need an extra ρ_+. Specifically,

$$\rho_{L\pm}\rho_{R\pm}[\mathbf{T}^2] = \rho_+\mathbf{T}^2\rho_+$$

is a lepton doublet, consisting of 2 Dirac spinors, one for the electron, one for its neutrino. (The particle identifications are not arbitrary. See particularly [2, Section 3.2] for the mathematics behind that statement.) Interestingly, this further reduction does not result in any further reduction of the 1-vector space of our original Clifford algebra, $\mathcal{CL}(1, 9)$. We are still left with a version of 1-vectors for $\mathcal{CL}(1, 3)$. However, the story is different for the space of 2-vectors. Initially they form a representation of the 1,9-Lorentz Lie algebra, $so(1, 9)$. After the initial reduction we get something more than $so(1, 3)$:

$$\rho_{L+}so(1, 9)\rho_{L+} = (so(1, 3) \times so(6))\rho_{L+},$$

and after the second spinor reduction,

$$\rho_{R+}\rho_{L+}so(1, 9)\rho_{L+}\rho_{R+} = (so(1, 3) \times u(1) \times su(3))\rho_{L+}\rho_{R+}.$$

This is precisely what it seems, and precisely the part of $so(1, 9)$ we observe to function in our seeable part of the universe. (Isospin $SU(2)$ arises from $\mathbf{H}_R$ (see [1, Section 3.5]; [2, Chapter 3]; and [6] for an extension of these ideas). In short, $\mathbf{H}_R$ is isomorphic to $\mathbf{H}$; the elements of unit norm are the 3-sphere, $S^3 \simeq SU(2)$; and this $SU(2)$ commutes with the Clifford algebra for 1,9-spacetime developed above, so it is an internal symmetry with respect to that spacetime.)

The situation is more complicated than this (see [1], [2]), but the overriding point being made here is that the mathematics of $\mathbf{T}$ can be viewed as implying we exist in an observable universe that must be dominantly matter, or antimatter (if we accept that everything carrying nontrivial $SU(3)$ color charges is not directly observable by us, which in this context includes quarks, anti-quarks, and the extra 6 dimensions of spacetime, all of which involve the octonion units, $e_p,\ p = 1, 2, 3, 4, 5, 6$, which carry those charges). Acceptance of this notion has the potential to imply far more profound things about physics.

I consider this an elegant explanation of why we perceive our universe to be composed of matter. There are many (a great many) open questions that will not be resolved here. Quarks, as mentioned, are unseeable, but detectable. This color confinement is thought to be related to energy considerations of the strong force — but confinement it is. So the question arises: are the extra 6 (or more) dimensions of space detectable, and if so, what is the mechanism that hides them from us? Is the antimatter universe detectable, and what mechanism hides it? (Note: our observable universe has the occasional antimatter particle whizzing around. It is not being suggested that these should be unseeable, but that there is an antimatter universe out there (whatever "there" means) that we do not see.) And beyond this, what is really needed is a (much) deeper theory from which one might glean insights into these unresolved problems.

A penultimate note: in [1, Section 6.3] it was pointed out that the original model allowed algebraically for matter-antimatter mixing via the extra 6 dimensions, but that reasonable conditions put on the dependence of the various particle fields on these extra dimensions led to these mixing pathways disappearing. Whatever the case, this idea of mixing is mediated by those extra 6 dimensions, which provide channels from the matter part of the overall universe to the antimatter part. Were these channels viable they would allow, for example, an electron from our matter part to channel through to the antimatter part, appearing on the other side as an antiquark (it necessarily picks up an anti-color charge en route). But this idea just scratches the surface.

And finally, I would like to add that this exploitation of $\mathbf{T}$ as the foundation of a model of reality is not the only one, it is the one I like best (well, I've been at it for over 30 years, so changing now is not going to happen). For an alternate approach, see [9].

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