Petra Smolíková

Homomorphism duality for rooted oriented paths

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Abstract: Let (H, r) be a fixed rooted digraph. The (H, r)-coloring problem is the problem of deciding for which rooted digraphs (G, s) there is a homomorphism $f : G \to H$ which maps the vertex s to the vertex r. Let (H, r) be a rooted oriented path. In this case we characterize the nonexistence of such a homomorphism by the existence of a rooted oriented cycle (C, q), which is homomorphic to (G, s) but not homomorphic to (H, r). Such a property of the digraph (H, r) is called rooted cycle duality or $*$-cycle duality. This extends the analogical result for unrooted oriented paths given in [6]. We also introduce the notion of comprimed tree duality. We show that comprimed tree duality of a rooted digraph (H, r) implies a polynomial algorithm for the (H, r)-coloring problem.

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