Michael van Hartskamp, Jan van Mill
Some examples related to colorings

Abstract: We complement the literature by proving that for a fixed-point free map $f : X \to X$ the statements (1) f admits a finite functionally closed cover $\mathcal{A}$ with $f[A] \cap A = \emptyset$ for all $A \in \mathcal{A}$ (i.e., a coloring) and (2) βf is fixed-point free are equivalent.
When functionally closed is weakened to closed, we show that normality is sufficient to prove equivalence, and give an example to show it cannot be omitted.
We also show that a theorem due to van Mill is sharp: for every $n \geq 2$ we construct a strongly zero-dimensional Tychonov space X and a fixed-point free map $f : X \to X$ such that f admits a closed coloring, but no coloring has cardinality less than n.

1