A. De Simone, M. Navara, P. Pták

On interval homogeneous orthomodular lattices

Comment.Math.Univ.Carolinae 42,1 (2001) 23-30.

Abstract: An orthomodular lattice L is said to be interval homogeneous (resp. centrally interval homogeneous) if it is σ-complete and satisfies the following property: Whenever L is isomorphic to an interval, $[a,b]$, in L then L is isomorphic to each interval $[c,d]$ with $c \leq a$ and $d \geq b$ (resp. the same condition as above only under the assumption that all elements a, b, c, d are central in L).
Let us denote by Inthom (resp. Inthom$_c$) the class of all interval homogeneous orthomodular lattices (resp. centrally interval homogeneous orthomodular lattices). We first show that the class Inthom is considerably large — it contains any Boolean σ-algebra, any block-finite σ-complete orthomodular lattice, any Hilbert space projection lattice and several other examples. Then we prove that L belongs to Inthom exactly when the Cantor-Bernstein-Tarski theorem holds in L. This makes it desirable to know whether there exist σ-complete orthomodular lattices which do not belong to Inthom. Such examples indeed exist as we than establish. At the end we consider the class Inthom$_c$. We find that each σ-complete orthomodular lattice belongs to Inthom$_c$, establishing an orthomodular version of Cantor-Bernstein-Tarski theorem. With the help of this result, we settle the Tarski cube problem for the σ-complete orthomodular lattices.

Keywords: interval in a σ-complete orthomodular lattice, center, Boolean σ-algebra, Cantor-Bernstein-Tarski theorem
AMS Subject Classification: 06C15, 06E05, 81P10