A.V. Arhangel'skii, L. Ludwig

On α-normal and β-normal spaces

Comment.Math.Univ.Carolinae 42,3 (2001) 507-519.

Abstract: We define two natural normality type properties, α-normality and β-normality, and compare these notions to normality. A natural weakening of Jones Lemma immediately leads to generalizations of some important results on normal spaces. We observe that every β-normal, pseudocompact space is countably compact, and show that if X is a dense subspace of a product of metrizable spaces, then X is normal if and only if X is β-normal. All hereditarily separable spaces are α-normal. A space is normal if and only if it is κ-normal and β-normal.
Central results of the paper are contained in Sections 3 and 4. Several examples are given, including an example (identified by R.Z. Buzyakova) of an α-normal, κ-normal, and not β-normal space, which is, in fact, a pseudocompact topological group. We observe that under CH there exists a locally compact Hausdorff hereditarily α-normal non-normal space (Theorem 3.3). This example is related to the main result of Section 4, which is a version of the famous Katětov's theorem on metrizability of a compactum the third power of which is hereditarily normal (Corollary 4.3). We also present a Tychonoff space X such that no dense subspace of X is α-normal (Section 3).

Keywords: normal, α-normal, β-normal, κ-normal, weakly normal, extremally disconnected, $C_p(X)$, Lindelöf, compact, pseudocompact, countably compact, hereditarily separable, hereditarily α-normal, property wD, weakly perfect, first countable
AMS Subject Classification: 54D15, 54D65, 54G20