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Countable compactness and p-limits

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Abstract: For $\emptyset \neq M \subseteq \omega^*$,we say that X is quasi M-compact, if for every $f : \omega \to X$ there is $p \in M$ such that $\overline{f}(p) \in X$, where $\overline{f}$ is the Stone-Čech extensionof f. In this context, a space X is countably compact iff$X$ is quasi ω^*-compact. If X is quasi M-compactand M is either finite or countable discrete in ω^*,then all powers of X are countably compact. Assuming CH,we give an example of a countable subset $M \subseteq \omega^*$ and a quasi M-compact space X whose square is notcountably compact, and show that in a model of A. Blass and S.Shelah every quasi M-compact space is p-compact (= quasi$\{p\}$-compact) for some $p \in \omega^*$, whenever $M \in [\omega^*]^{<\mathfrak{c}}$. We prove that if $\emptyset \notin \{T_\xi : \xi < 2^{\mathfrak{c}}\} \subseteq [\omega^*]^{<2^{\mathfrak{c}}}$, then there is a countably compact space X that is not quasi T_ξ-compact for every $\xi < 2^{\mathfrak{c}}$; hence, if $2^{\mathfrak{c}}$ is regular, then there is a countably compact space X such that X is not quasi M-compact for any $M \in [\omega^*]^{<2^{\mathfrak{c}}}$. We list some open problems.

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