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Weighted Miranda–Talenti inequality and applications to equations with discontinuous coefficients

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Abstract: Let Ω be an open bounded set in $\mathbb{R}^n$ $(n \geq 2)$, with C^2 boundary, and $N^{p,\lambda}(\Omega)$ $(1 < p < +\infty,\ 0 \leq \lambda < n)$ be a weighted Morrey space.
In this note we prove a weighted version of the Miranda-Talenti inequality and we exploit it to show that, under a suitable condition of Cordes type, the Dirichlet problem:

$$\begin{cases} \sum_{i,j=1}^n a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} = f(x) \in N^{p,\lambda}(\Omega) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

has a unique strong solution in the functional space

$$\left\{ u \in W^{2,p} \cap W_o^{1,p}(\Omega) : \frac{\partial^2 u}{\partial x_i \partial x_j} \in N^{p,\lambda}(\Omega), \quad i,j = 1,2,\ldots,n \right\}.$$

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