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Products of Lindelöf T_2-spaces are Lindelöf — in some models of ZF

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Abstract: The stability of the Lindelöf property under the formation of products and of sums is investigated in ZF (= Zermelo-Fraenkel set theory without AC, the axiom of choice). It is

- not surprising that countable summability of the Lindelöf property requires some weak choice principle,
- highly surprising, however, that productivity of the Lindelöf property is guaranteed by a drastic failure of AC,
- amusing that finite summability of the Lindelöf property takes place if either some weak choice principle holds or if AC fails drastically.

Main results:

1. Lindelöf = compact for T_1-spaces iff CC($\mathbb{R}$), the axiom of countable choice for subsets of the reals, fails.
2. Lindelöf T_1-spaces are finitely productive iff CC($\mathbb{R}$) fails.
3. Lindelöf T_2-spaces are productive iff CC($\mathbb{R}$) fails and BPI, the Boolean prime ideal theorem, holds.
4. Arbitrary products and countable sums of compact T_1-spaces are Lindelöf iff AC holds.
5. Lindelöf spaces are countably summable iff CC, the axiom of countable choice, holds.
6. Lindelöf spaces are finitely summable iff either CC holds or CC($\mathbb{R}$) fails.
7. Lindelöf T_2-spaces are T_3 spaces iff CC($\mathbb{R}$) fails.
8. Totally disconnected Lindelöf T_2-spaces are zerodimensional iff CC($\mathbb{R}$) fails.

Keywords: axiom of choice, axiom of countable choice, Lindelöf space, compact space, product, sum
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