Eugene D. Rodionov, Viktor V. Slavskii Conformal deformations of the Riemannian metrics and homogeneous Riemannian spaces

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Abstract: In this paper we investigate one-dimensional sectional curvatures of Riemannian manifolds, conformal deformations of the Riemannian metrics and the structure of locally conformally homogeneous Riemannian manifolds. We prove that the nonnegativity of the one-dimensional sectional curvature of a homogeneous Riemannian space attracts nonnegativity of the Ricci curvature and we show that the inverse is incorrect with the help of the theorems O. Kowalski-S. Nikčević [K-N], D. Alekseevsky-B. Kimelfeld [A-K]. The criterion for existence of the left-invariant Riemannian metrics of positive one-dimensional sectional curvature on Lie groups is presented. Classification of the conformally deformed homogeneous Riemannian metrics of positive sectional curvature on homogeneous spaces is obtained. The notion of locally conformally homogeneous Riemannian spaces is introduced. It is proved that each such space is either conformally flat or conformally equivalent to a locally homogeneous Riemannian space.

 $\mathbf{Keywords:}$ conformal deformations, Riemannian metrics, homogeneous Riemannian

spaces

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