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*On the composition of the integral and derivative operators of functional order*

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**Abstract:** The Integral,  $I_\phi$ , and Derivative,  $D_\phi$ , operators of order  $\phi$ , with  $\phi$  a function of positive lower type and upper type less than 1, were defined in [HV2] in the setting of spaces of homogeneous-type. These definitions generalize those of the fractional integral and derivative operators of order  $\alpha$ , where  $\phi(t) = t^\alpha$ , given in [GSV].

In this work we show that the composition  $T_\phi = D_\phi \circ I_\phi$  is a singular integral operator. This result in addition with the results obtained in [HV2] of boundedness of  $I_\phi$  and  $D_\phi$  or the  $T1$ -theorems proved in [HV1] yield the fact that  $T_\phi$  is a Calderón-Zygmund operator bounded on the generalized Besov,  $\dot{B}_p^{\psi,q}$ ,  $1 \leq p, q < \infty$ , and Triebel-Lizorkin spaces,  $\dot{F}_p^{\psi,q}$ ,  $1 < p, q < \infty$ , of order  $\psi = \psi_1/\psi_2$ , where  $\psi_1$  and  $\psi_2$  are two quasi-increasing functions of adequate upper types  $s_1$  and  $s_2$ , respectively.

**Keywords:** fractional integral operators, fractional derivative operators, spaces of homogeneous type, Besov spaces, Triebel-Lizorkin spaces

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